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The length and severity of the coronavirus recession estimated from the dynamics of relaxing lockdown

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Abstract
The J-value is used to cast light on the policy of lockdown and its relaxation by linking findings from epidemiological and economic analysis. A new, two-cohort model is applied to the coronavirus epidemic to allow for different sections of the population exhibiting different responses to the same disease. Optimal parameter sets are generated by adjusting the model to fit the data on new daily cases in England up to 10 April 2020. While several sets of parameter values give an equally good match to the data, a common feature is that the disease transients for the controlled second wave of the disease are long-drawn out, with tails resulting from an easing of restrictions in May 2020 lasting to the end of 2021 or later. A further feature in common is the lack of population or herd immunity at the end of the second wave, which leaves the population of England, and the UK by extension, open to further significant waves of the infection.

While considerable uncertainties remain with the epidemiology, there is no doubt that the positive feedback nature of the Covid-19 epidemic will make controlling the present outbreak a very difficult task, especially if the main regulating tool selected is the co-ordinated behaviour of 67 million people. The likely economic effect is analysed of the years-long process to move out of lockdown sufficiently slowly so as not to cause excessive strain on the health services. In the Extended Base Case, annual GDP will fall by 23.5% in 2020 and is not likely to recover to pre-lockdown levels until 2024. Applying the J-value derived Bristol Curve of population-average life-expectancy versus GDP suggests that the strategy of restriction followed by gradual relaxation is likely to result in a net cost, in terms of average human lives lost, that will be comparable with the UK's sacrifice over the six years of World War Two. A policy of lockdown followed by gradual relaxation is likely to do much more harm to the nation's health than good.

Key words: Coronavirus, Covid-19, lockdown, lockdown exit strategy, J-value, multi-cohort epidemic model, economic challenge

1. Introduction

Using a previous J-value[1] result, a recent study showed that a countermeasure against Covid-19 could sacrifice more life than it gained if it led to a fall in economic output that was greater than 6.4% per person and the downturn continued for a prolonged period[2]. The financial crash in the first decade of the 21st century provided a close parallel for a similar decrease. Here there had been a drop of 6% in UK Gross Domestic Product (GDP) per head between 2007 and 2009, with full recovery not occurring until six years later. The nadir of the recession was followed two years later by the stalling in the growth of longevity. The steady, 2½ months improvement observed in UK life expectancy at birth for the previous 30 years came to an end in 2011 and has not recovered.

The Office for Budget Responsibility (OBR) reported, on 14 April 2020, that a coronavirus lockdown of three months, followed by a further three-month period when the restrictions were gradually lifted, would lead to a 35% fall in GDP in the second quarter of 2020, with the annual fall suggested as being 13%[3]. However, the OBR assumed a rapid bounce back, an assumption on which other commentators have cast doubt. For example, Chris Giles wrote in the Financial Times: "The OBR's second assumption was that the recovery will be rapid and total — that there will be almost no hangover from the crisis. Very few people believe this."[4] In a similar vein, the IFO Institute predicted that a two-month period of confinement would cause the UK's annual GDP to decline by between 7.7% and 13% while a three-month partial shutdown of the economy would lead to a fall in 2020's GDP of 10.7 to 19%.[5]

The UK Government announced, on 16 April 2020, that the three-week lockdown it had brought into force on 23 March 2020 would be extended by a further 3 weeks to the first week of May 2020, although no firm decision on a return to work on this date had been made. This partial elucidation of the Government's strategy invites an examination of the case where a return to work begins 6 weeks after the beginning of the lockdown, but with extensive restrictions remaining in place.

The additional information that has become available in the month since the first, related study[2] was carried out make it clear that many people in the UK who have contracted Covid-19 are not tested, so that their cases are never confirmed nor entered into the official records. For example, the Prime Minister was tested on 27 March 2020 and found to be suffering from the SARS-CoV-2 virus. He was admitted to hospital on 5

[1] Thomas, P. and Waddington, I., 2017,"Validating the J-value safety assessment tool against pan-national data", *Process Safety and Environmental Protection*, Vol. 112A, 179 – 197, November.
[2] Thomas, P., 2020, "J-value assessment of how best to combat COVID-19", *Nanotechnology Perceptions*, Vol.16, 16–40.
[3] Office of Budget Responsibility, 2020, *Commentary on the OBR coronavirus reference scenario*, 14 April, https://cdn.obr.uk/Coronavirus_reference_scenario_commentary.pdf
[4] Giles, C., 2020, The OBR's assumption that life will return to normal is optimistic, Financial Times, 16 April, https://www.ft.com/content/73a35c9e-7f41-11ea-82f6-150830b3b99a
[5] Dorn, F. et al., 2020, The economic costs of the coronavirus shutdown for selected European countries: a scenario calculation, (EconPol Policy Brief 25), Munich, IFO Institute.

April 2020, discharged after a week, which included three days in an intensive care unit, and then spent two weeks recovering at Chequers[6] before returning to work on 25 April. Meanwhile his fiancée self-isolated in her own home away from Downing Street on about the same date because she believed that she too was suffering from coronavirus. She recovered without medical intervention after a week in bed[7]. But, because she was not tested, her case will never be included in the official record even though it is highly probable that she was infected with the same virus.

The fraction of cases of Covid-19 going unrecorded is not known, but it is likely to coincide roughly with the percentage of people who experience either mild or no symptoms, that is to say about 80% of the population[8]. Whereas simulation of the Covid-19 outbreak in the previous paper[2] applied a single-cohort model based on the official number for infections reported in the UK, the mathematics have now been extended to encompass a multi-cohort model. This allows for different groups in the population responding in different ways after contracting the disease. The basic reproduction number and the average time between successive generations are permitted to vary from one population cohort to the next.

The main vehicle used in this study is a two-cohort model, with the smaller group made up of those who, if infected, would display symptoms severe enough to warrant testing under the *de facto* UK conventions in force to 10 April 2020, when testing capacity was limited. The larger group is comprised of those people who will experience either mild or no symptoms if they are infected with the coronavirus.

The research will assess first how far the lockdown has been successful in reducing the spread of Covid-19 and then how far it is possible to ease restrictions before provoking such an increase in cases that the National Health Service (NHS) becomes overwhelmed. Avoiding putting excessive strain on the resources of the UK's principal medical service was the reason advanced by the Government for the lockdown, as embodied in its exhortation, "Stay at home, protect the NHS, save lives".[9]

The calculated transients of new cases and the estimated extent of the residual restrictions that need to be kept in place then allow an assessment to be made of the length and the severity of the coronavirus-related recession in the UK under the strategy of restriction.

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[6] Guardian staff, 2020, PM's Covid-19 timeline: from 'mild symptoms' to a brush with death, 12 April, The Guardian, https://www.theguardian.com/world/2020/apr/05/timeline-boris-johnson-and-coronavirus

[7] Saunt, R. and Howard, H., 2020, "Carrie Symonds has coronavirus too: Boris Johnson's pregnant fiancée reveals she has been in bed for seven days with symptoms of disease that has also put the Prime Minister into isolation", Daily Mail, 4 April, https://www.dailymail.co.uk/news/article-8187653/Carrie-Symonds-got-coronavirus-Boris-Johnsons-pregnant-fiancee-admits-tested-positive.html

[8] Worldometers, 2020, COVID-19 Coronavirus / Symptoms, accessed 22 April https://www.worldometers.info/coronavirus/coronavirus-symptoms/#mild

[9] Department of Health and Social Care, Coronavirus: stay at home, protect the NHS, save lives - web version, Updated 15 April 2020 https://www.gov.uk/government/publications/coronavirus-covid-19-information-leaflet/coronavirus-stay-at-home-protect-the-nhs-save-lives-web-version

The layout of the paper is as follows. Following an Introduction that sets out the context for the research, Section 2 summarises the two-cohort model. Section 3 explains how the model is fitted to the transient data on new laboratory-confirmed daily cases. Section 4 lays out how the emergence from lockdown is programmed.

Section 5 presents results in the form of tables and commentary for a range of assumptions on two key variables: (i) the fraction of people in the population who are sensitive to the coronavirus and would become sufficiently ill if they contracted Covid-19 to be tested under the *de facto* testing conventions applying to 10 April 2020 in the UK and (ii) the "easing efficiency", which specifies the fraction of the effectiveness of the full lockdown that is maintained after each stage of partial relaxation. The first variable is not known with any precision, but it is assumed to lie somewhere near the fraction, 20%, of people reported to suffer more severe symptoms. The easing efficiency may be regarded as a control variable in the strategy of relaxing restrictions, which may be varied by the Government in steps over time from 1.0 at lockdown to 0.0 when all restrictions are lifted. It will be emphasised that no deterministic forecast of the effects of easing is possible because of there appears to be no unique solution to the model matching process given the current, limited level of knowledge of the disease. Several solutions with parameters within broadly credible ranges give equally good matches to the data. However a number of commonalities emerge in the reponse of the epidemic that will prove useful in the assessment of the likely severity and duration of the coronavirus recession associated with lockdown followed by gradual relaxation of restrictions. A Base Case scenario is established with parameters that are not too far from those used in the first study[2] that gives outcomes that are roughly mid-range in their severity.

Section 6 extends the Base Case through to 2025, during which time restrictions are assumed to be relaxed carefully and at intervals to the point where they are removed altogether. The sizes of the 3rd and 4th waves are calculated. The developing population or "herd" immunity is tracked, and then finally the likely number of coronavirus-related deaths is estimated.

Section 7 uses the Extended Base Case as a foundation for calculating the effect on GDP, using an economic model for which details are given in Appendix C. These results are then translated into the nation's likely loss of life expectancy as a result of recession. The amount of life that is lost is then compared with the possible saving in life as a result of keeping the NHS from being over-stressed by spreading the epidemic over 5 years.

Section 8 provides a discussion, while Section 9 comments on the limitations of the data and the model. Conclusions are given in Section 10.

Appendix A derives a multi-cohort model, a two-cohort version of which is applied in this study. For completeness, Appendix B shows the conditions governing the degeneration of the multi-cohort model degenerating to the single-cohort version used in the previous research[2].

Appendix C derives a model for the effect of varying degrees of lockdown on weekly production using results provided by the OBR[3]. Estimates are made of the UK's annual GDP for the years 2020 to 2024 inclusive when a policy of restriction is applied as exemplified by the Extended Base Case.

2. Summary of the two-cohort model

The derivation of the multi-cohort model is given in Appendix A, a two-cohort version of which is used to model the Covid-19 epidemic. People in cohort 1 are expected to experience significant symptoms if they contract SARS-CoV-2. Once they fall ill, they will be tested for the active virus and found positive. Some but not all of them will be admitted to hospital. Meanwhile the members of the second cohort will experience less severe symptoms or else be asymptomatic and will never be subjected to a viral test.

Applying equation (A.8) from Appendix A gives the growth in the time-varying number, $n_1(t)$, of infectious people in cohort 1 as:

$$\frac{dn_1}{dt}(t) = \frac{n_{s,1}(t)}{N}\left(R_{01}\frac{n_1(t)}{\tau_{\text{inf},1}} + R_{02}\frac{n_2(t)}{\tau_{\text{inf},2}}\right) - \frac{n_1(t)}{\tau_{\text{inf},1}} \qquad (1)$$

where $n_{s,1}(t)$ is the time-varying number of susceptible people in cohort 1, N is the population of England (56,000,000, 84% of the UK's total[10]), R_{01} is the basic reproduction number for cohort 1, $\tau_{\text{inf},1}$ is the average time between generations for cohort 1, $n_2(t)$ is the time-varying number of infectious people in cohort 2, and $\tau_{\text{inf},2}$ is the average time between generations for cohort 2.

The corresponding expression for the number of infectious people in cohort 2 will take the form:

$$\frac{dn_2}{dt}(t) = \frac{n_{s,2}(t)}{N}\left(R_{01}\frac{n_1(t)}{\tau_{\text{inf},1}} + R_{02}\frac{n_2(t)}{\tau_{\text{inf},2}}\right) - \frac{n_2(t)}{\tau_{\text{inf},2}} \qquad (2)$$

where $n_{s,2}(t)$ is the time-varying number of susceptible people in cohort 2.

From equation (A.6), the time-varying number, $n_{r1}(t)$, of people in cohort 1 who have recovered or died will be

[10] World Population Review, 2020, England population 2020, https://worldpopulationreview.com/countries/england-population/

$$\frac{dn_{r1}}{dt}(t)=\frac{n_1(t)}{\tau_{\text{inf},1}} \qquad (3)$$

while the corresponding number, $n_{r2}(t)$ from cohort 2 will be

$$\frac{dn_{r2}}{dt}(t)=\frac{n_2(t)}{\tau_{\text{inf},2}} \qquad (4)$$

Using equation (A.3), the time-varying numbers of susceptible people in each cohort will be:

$$n_{s1}(t)=\theta_1 N-n_1(t)-n_{r1}(t) \qquad (5)$$

and

$$n_{s2}(t)=\theta_2 N-n_2(t)-n_{r2}(t) \qquad (6)$$

where θ_1 is the fraction of the population assumed to make up cohort 1, while $\theta_2=1-\theta_1$ is the fraction making up cohort 2.

The four differential equations, (1) to (4), may be integrated numerically from initial conditions, $n_1(0), n_2(0) . n_{r1}(0)$ and $n_{r2}(0)$ to map the dynamic progress of the epidemic. The algebraic equations (5) and (6) are then evaluated at each time step.

Two of the initial conditions will be zero:

$$n_{r,1}(0)=n_{r,2}(0)=0 \qquad (7)$$

The initial condition for the number of infections people in cohort 1 will be $n_1(0)=1$, the figure given in the official records for the nominal start date, 30 January 2020, of the UK's epidemic. However the number of people in cohort 2, $n_2(0)$, who are infectious with the coronavirus on that date, but whose cases will never be confirmed by a viral test, is not known. This becomes a free parameter open to adjustment in the optimisation process.

3. Fitting the two-cohort model to the recorded data

The number of people testing positive for coronavirus each day is a leading indicator that is published each day for England,[11] subject to the caveat, "only data from five days or more ago can be considered complete"[12]. It was chosen to match the model to the

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[11] GOV.UK, 2020, Coronavirus (COVID-19) in the UK, https://coronavirus.data.gov.uk/

time series of daily new positive tests starting on 30 January 2020, the date of the first confirmed case in the UK, through to 10 April 2020 inclusive. The data points were taken from the Government website on 18 April 2020 to allow the numbers to settle.

The effect of the lockdown's measures may be assumed to have come into force at 00.01 on the day 24 March 2020 (the lockdown date, t_L) after it was announced. It is further assumed that the general public became gradually more proficient at conforming over the next three weeks (to midnight on 13 April 2020). This renders the basic reproduction numbers time-varying, decreasing during the lockdown in line with the following expression:

$$R_{0i}(t) = \begin{cases} R_{0i}(0) & t < t_L \\ \left(1 - \frac{f_{\Delta R0}}{21}(t - t_L)\right) R_{0i}(0) & t_L \le t < t_L + 21 \\ (1 - f_{\Delta R0}) R_{0i}(0) & t \ge t_L + 21 \end{cases} \quad \text{for } i = 1, 2 \tag{8}$$

where $f_{\Delta R0}$ is the fractional decrease in the two basic reproduction numbers brought about by the lockdown.

A positive test can be expected to lag on infection by the time, about 5 days, it takes for symptoms to be experienced. This could put the start of the decline in the basic reproduction numbers back into the previous week, just after the Prime Minister asked the public to observe voluntary social distancing measures on 16 March 2020. Professor Carl Henegan has argued for the voluntary measures called for in that week having a significant effect in curbing the numbers of coronavirus deaths.[13]

By the definition of cohort 1, the model's predicted number of new positively tested cases is the differential, dn_{x1}/dt, provided by equation (A.4) with $i = 1$:

$$\frac{dn_{x1}}{dt}(t) = \frac{n_{s,1}(t)}{N}\left(R_{01}\frac{n_1(t)}{\tau_{\text{inf},1}} + R_{02}\frac{n_2(t)}{\tau_{\text{inf},2}}\right) \tag{9}$$

Given a value of the fraction, θ_1, of the population belonging to cohort 1, the sum of the squared errors between the reported numbers of new positive tests and dn_{x1}/dt may be minimised by an optimal choice of values for the 6 parameters:

$$R_{01}, \tau_{\text{inf},1}, R_{02}, \tau_{\text{inf},2}, f_{\Delta R0} \text{ and } n_2(0)$$

[12] GOV.UK, 2020, About the data, https://coronavirus.data.gov.uk/about
[13] Boyd, C., Blanchard, S. and Matthews, S., 2020, UK announces 449 more coronavirus deaths - the fewest for a fortnight as leading expert argues Britain's crisis peaked BEFORE lockdown and claims fatality rate could be as low as 0.1%, Daily Mail, 20 April, https://www.dailymail.co.uk/news/article-8235979/UKs-coronavirus-crisis-peaked-lockdown-Expert-argues-draconian-measures-unnecessary.html

The hospitalisation fraction, f_{hosp}, is calculated as follows. It is assumed in the Base Case that 20% of the population has the potential for significant symptoms; these people would be subject, after showing signs, to test under the conventions applied up to 10 April 2020 and would then emerge with a positive diagnosis. If all the population were to contract the disease, then the number hospitalised would be $f_{hosp} \times 0.2N$. The European Centre for Disease Prevention and Control reported on 8 April 2020 that the crude fatality rate amongst hospitalised patients in the EU/EEFA countries was 11%[14]. Assuming that figure is representative of England, then the number of people who would die would be $0.022 f_{hosp} N$. Taking the death rate for the population as a whole, if everyone were infected, as 0.99%[2], then equating numbers of deaths gives $0.022 f_{hosp} N = 0.099N$. Hence the hospitalisation rate for England emerges as $f_{hosp} = 0.99/2.2 = 0.45$. This rate is higher than the average of 32% of diagnosed cases in the EU/EEFA countries requiring hospitalisation, but it may well be representative given the UK's well publicised shortage of viral testing kits in the early stages of the epidemic.

4. Programming the partial emergence from lockdown

It is assumed that the relaxation of the lockdown measures will reduce the protective effect they achieved during lockdown. In full lockdown, the effective basic reproduction numbers will be $\left(1-f_{\Delta R0}\right)R_{0i}$, i = 1,2, whereas after relaxation, the effect basic reproduction numbers will be $\left(1-\eta f_{\Delta R0}\right)R_{0i}$, i = 1,2. Here $\eta : 0 \le \eta \le 1.0$ will be named the "easing efficiency". By definition, a 100% easing efficiency would retain all of the social distancing benefits of lockdown but, in reality, the effective basic reproduction numbers after any degree of relaxation will be somewhat closer to their pre-lockdown values.

Assuming that the partial opening up is introduced gradually over a period of three weeks, then the basic reproduction numbers will obey the relation:

$$R_{0i}\left(t\right) = \begin{cases} \left(1-f_{\Delta R0}\right)R_{0i}\left(0\right) & t < t_R \\ \left(1-f_{\Delta R0}+\left(1-\eta\right)\dfrac{f_{\Delta R0}}{21}\left(t-t_R\right)\right)R_{0i}\left(0\right) & t_R \le t < t_R + 21 \\ \left(1-\eta f_{\Delta R0}\right)R_{0i}\left(0\right) & t \ge t_R + 21 \end{cases} \quad \text{for } i = 1,2 \qquad (10)$$

where T_R is the date, 00.01 on 5 May 2020, when restrictions are assumed to start to be relaxed.

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[14] European Centre for Disease Prevention and Control, 2020, Rapid risk assessment: Coronavirus disease 2019 (COVID-19) pandemic: increased transmission in the EU/EEFA and UK –eighth update, 8 April, https://www.ecdc.europa.eu/en/publications-data/rapid-risk-assessment-coronavirus-disease-2019-covid-19-pandemic-eighth-update

5. Results

A notable feature of the matching process is that no unique set of values emerges from the optimisation to find the values of $R_{01}, \tau_{\text{inf},1}, R_{02}, \tau_{\text{inf},2}, f_{\Delta R0}$ and $n_2(0)$ that produce the closest match to the reported data. That this should be the case even with the simple, 4-state model used here backs up concerns[15] on the use of highly detailed epidemiological models when, as in the current situation, there is significant uncertainty on the characteristics of the disease. Such caution may be advisable more generally when pandemic modelling is used to inform coping strategies, since the illness to be controlled will very often be new and poorly understood at the time when decisions are needed. Models that require fewer parameters to be derived from the data are likely to be more robust, easier to validate against evidence and give guidance that is easier to understand.

The approach taken here is first to assign a value to the easing efficiency, η. Then data-sets are found that allow the model to match the transient closely but where the values for the basic reproduction numbers, R_{0i}, and the average times, $\tau_{\text{inf},i}$, between generations for both cohorts lie reasonably close to the corresponding figures used in the previous study[2], namely $R_0 = 2.35$ and $\tau_{\text{inf}} = 9.5$ days. However, other sets of parameter values that provide a very good match to the transient data are possible.

The time series of new daily cases is readily available only for England, and so the results apply to that country only. A rough conversion to the whole of the United Kingdom may be made by scaling up the quantities in proportion to the populations of the UK (67 million) and England (56 million), a factor of about 1.2.

5.1 Allowing for differences in characteristics between the two cohorts

5.1.1 Cohort 1 contains 20% of the population: θ_1 = 0.2, easing efficiency is 80%: η = 0.8

Here it is assumed that 20% of the people in the country have the potential for significant symptoms; they would be subject, after showing signs, to test under the conventions applied up to 10 April 2020[16] and would then emerge with a positive diagnosis. This fraction of the population will make up cohort 1, so that $\theta_1 = 0.2$. The remaining 80% will be members of cohort 2.

It is assumed, in evaluating behaviour after 4 May 2020 that the relaxation is conducted carefully enough to retain 80% of the benefit of the social distancing practised during lockdown. Hence the easing efficiency, η, is assigned the value, 0.8.

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[15] Ramsden, J. J., 2020, Editorial: COVID-19, Nanotechnology Perceptions, Vol. 16, 5 – 15.

[16] This caveat is added because the UK Government is seeking to expand greatly the number of viral tests made every day. Hence the testing conventions in place to 10 April 2020, which were based on limited availability of tests for the general public, are likely to change. In particular, more people from the part of the population notionally assigned to Cohort 2 could be tested.

Scenario (i): Base Case

Figure 1 shows the best match between model prediction and the reported number of new tested and confirmed cases each day. The root mean square of the differences between model and reported values is just under 212 cases, about 5% of the highest readings, where the data reports show most volatility. The model predicts a peak in reported confirmed cases in England between the 5th and 6th of April 2020.

Figure 2 shows the post-relaxation transient of daily positive tests made under the UK's conventions to 10 April 2020. The confirmed cases are predicted to rise past 4,000 per day by mid-June and to continue to grow fairly steadily over the next 3 months to a peak of about 16,500 new daily cases by the end of October 2020. At this stage the number of immune people in the country will have reached a level where transmission can no longer grow and a long fall in new cases will begin. The new daily number confirmed cases per day (under the testing conventions in force in the UK to 10 April 2020) will fall below 4,000 by the end of March 2021. The number of new daily cases in England is predicted to drop below 100 by the middle of October 2021, signalling that the second outbreak is nearly over.

It is clear that the second wave is larger than the first, with the peak of new daily cases about four times the roughly 4,000 recorded in the first surge.

The maximum hospital bed occupancy required for Covid-19 patients in the second wave is about 61,000. For comparison, there were reported to be 42,540 beds unoccupied in the NHS on 23 April 2020.[17]

Table 1(i) shows the best-fit parameters and some key variables characterising the behaviour of the second wave. The total number of deaths by the end of the 2nd wave is 172,000. The previous study[2] estimated the mean number of years remaining to the victims as between 15 and 17 years, based on them enjoying average health for their age before becoming infected with the coronavirus. However data from the Office of National Statistics has shown that 91% of people dying from Covid-19 had at least one pre-existing condition, such as ischaemic heart disease, pneumonia, dementia and chronic obstructive pulmonary disease (COPD), with an average of 2.7 such conditions.[18] Recent research taking account of co-morbity lowered the likely average loss of life expectancy to about 12 years.[19] This suggests a total loss of life of 49,000 plex-2020, where the population-average life expectancy (plex-2020) is the amount of life, 42 years, that would

[17] Nelson, F., 2020, Contact tracing is no silver bullet but it may help to end the lockdown, The Daily Telegraph, 24 April.
[18] Office of National Statistics, 2020, Deaths involving COVID-19, England and Wales: deaths occurring in March 2020, 16 April.
https://www.ons.gov.uk/peoplepopulationandcommunity/birthsdeathsandmarriages/deaths/bulletins/deathsinvolvingcovid19englandandwales/deathsoccurringinmarch2020
[19] Hanlon, P., et al., 2020, COVID-19 – exploring the implications of long-term condition type and extent of multimorbidity on years of life lost: a modelling study, [awaiting peer review], Wellcome Open Research, https://wellcomeopenresearch.org/articles/5-75 .

be given up by an average UK citizen if involved in an immediately fatal accident in 2020.

31% of the population would have gained at least temporary immunity by the end of 2021, but this falls short of the 60% or more required to give population or "herd" immunity in the unrestricted state where no social distancing measures were imposed or advised. The population of England and, by extension, the UK, would remain vulnerable to another epidemic of Covid 19 if the country were to let up on its restrictions (at the level where easing efficiency is 80%) in the foreseeable future unless or until a vaccine or a cure becomes available.

<u>Scenario (ii)</u>

This variant on the Base Case of Scenario (i) is characterised by Table 1(ii). The principal difference from Scenario (i) lies in the smaller number of hospital beds required to accommodate the second wave, down from 61,000 to 48,000, which results from the shorter average time between generations for cohort 1.

<u>Scenario (iii).</u>

This scenario is distinguished from the Base Case by having higher values for R_{0i} and $\tau_{\mathrm{inf},i}$, $i = 1, 2$. Numbers of infections are general higher, resulting in 58,000 more deaths, equivalent to an extra 15,000 plex-2020.

Health services will be put under greater strain because of the 60% increase over the Base Case in the maximum daily new cases to 26,000. Meanwhile the peak demand for hospital beds during the second wave more than doubles to nearly 133,000.

<u>General comments</u>

Scenarios (i), (ii) and (iii) fit the data on daily new infections to 10 April 2020 equally well. In each case the average person in cohort 1, which contains people who are more sensitive to the virus, is characterised by a higher degree of infectivity and a shorter average time between generations than apply to cohort 2.

Scenario (i) is labelled as the Base Case, but it is not argued that there are no other contenders with an equivalent claim to being representative.

5.1.2 Cohort 1 contains 20% of the population: θ_1 = 0.2, easing efficiency varies between 70% and 90%

<u>Scenario (i) Easing efficiency, $\eta = 0.7$</u>

Table 2 (i) gives the values characterising Scenario (i) of this subsection. The extra 10% drop in easing efficiency leads to the maximum daily confirmed cases rising to 43,000, more than double the Base Case. Peak hospital bed occupancy during the second wave is now 155,000 rather than the 61,000 of the Base Case, an increase of more than 150%.

The number of deaths by the end of the second wave is 260,000, which represents a loss of life expectancy of 74,000 plex-2020.

46% of the population would have been infected by the end of the second wave. This is not enough to achieve population or "herd" immunity, but the fraction of people who have had the disease has moved roughly three quarters of the way in that direction.

Scenario (ii) Easing efficiency, $\eta = 0.9$

The parameter values for Scenario (ii) of this subsection are given in Table 2(ii). The maximum number of daily confirmed cases (under the 10 April testing conventions) is 2,500, about a sixth of the figure for the Base Case and well below the peak that was experienced in early April 2020. Figure 3 traces the development of predicted daily positive tests over time, which shows only a small rise from the trough at the end of the lockdown period.

Under this scenario, deaths are kept down to 56,000, equivalent to 18,000 plex-2020 in lost life expectancy.

Only 5% of the population would have been infected by the end of the second wave.

General comments

It is clear that easing efficiency has a highly nonlinear effect on the severity of the second peak. Losing only 10% of the benefit of the lockdown (an easing efficiency of 90%) renders the second peak smaller than the first one.

Losing 20% of the advantage conferred by down (the assumption of the Base Case) means that the peak number of daily cases would rise by a factor of 4 over the maximum level during the first wave and the required hospital bed occupancy during the second wave would be roughly 50% more that the reported free hospital capacity on 23 April 2020.

A drop of 30% compared with lockdown would lead to the maximum number of daily cases being 10 times the highest level seen in the first wave. The peak number of hospital beds needed during the second wave would be nearly four times the spare capacity recorded on 22 April 2020.

5.1.3 Easing efficiency is 80%: η = 0.8; Cohort 1 contains between 10% and 30% of the population

Scenario (i) Cohort 1 population fraction, $\theta_1 = 0.1$

This scenario has outcomes at the lower end of the scale. The maximum rate of confirmed cases is 2,000 per day, and the maximum hospital bed occupancy during the second wave is 6,000, both substantially lower than were experienced in the first wave.

The assumption that the vulnerable population notionally contained in Cohort 1 makes up only 10% of the population rather than the 20% of the Base Case means that the implied death rate is halved to 0.49%. The number of deaths at the end of the second wave is 35,000, equivalent to 7,000 plex-2020 in terms of average lives lost, similar to the corresponding figure for Scenario (ii) of subsection 5.1.2.

Only an eighth of the population will have been infected by the end of the second wave. Clearly this is a long way from the roughly five eighths needed for herd immunity.

Scenario (ii) Cohort 1 population fraction, $\theta_1 = 0.3$

Now the maximum number of confirmed cases daily (April 10 testing conventions) is 23,000, about 5 times the peak seen in the first wave. The peak demand for hospital beds is, at 95,000 more than double the free capacity on 22 April 2020.

The assumption that the vulnerable population notionally contained in Cohort 1 makes up 30% of the population rather than the 20% of the Base Case means that the implied death rate is 50% higher at 1.48%. The number of deaths at the end of the second wave is 246,000, equivalent to 70,000 plex-2020 in terms of average lives lost. These figures are similar to those of Scenario (iii) of subsection 5.1.1 and of Scenario (i) of subsection 5.1.2.

About 30% of England's population is calculated to have been affected by the end of the second wave, halfway to herd immunity.

5.2 Special case where the two cohorts are constrained to share the same basic reproduction number, R_0, and the same average time between generations, τ_{inf}

Two scenarios are presented, in each of which Cohort 1 contains 20% of the population: $\theta_1 = 0.2$, while the easing efficiency is 80%: $\eta = 0.8$. Table 5(i) gives the parameter values defining Scenario (i) of this subsection, while Table 5(ii) provides the corresponding figures for Scenario (ii). These two scenarios give as good a match to the reported data on new cases daily as the other scenarios previously reported.

Scenario (i) has about half the severity of Scenario (ii).

Scenario (ii) of this subsection has similarities with Scenario (iii) of Section 5.1.1, as laid out in Table 1(iii).

6. The third and fourth waves: the Extended Base Case

The Base Case has been extended to 2025 to illustrate the likely duration required for social distancing measures under the policy of restriction.

Since only 31% of the population has been infected by the end of the second wave (taken as 31 July 2021), herd immunity in the unrestricted state has not been reached. However, the extra immunity achieved is useful as it allows a greater relaxation to be made in the next phase of the epidemic's control. Let the easing efficiency be lowered by a further 0.33 on 1 August 2021, so that now $\eta = 0.47$.

A third wave of new cases now occurs that is smaller than the second wave. The population immunity is 51% at the end of this phase, taken as 28 February 2023. The proximity of immunity at this stage to herd immunity in the unrestricted state means that all restrictions may be removed on 1 March 2023.

Table 4 summarises the dates of imposition and relaxation of the lockdown restrictions.

The resulting fourth wave is smaller than the third, and the entire epidemic may be regarded as essentially over by December 2024, roughly 5 years after the coronavirus reached the country. See Figure 4.

Two thirds of the population are calculated to have been infected with Covid-19 by the end of 2024.

The number of deaths in England, is 366,000 over the 5 year period. This is equivalent, in terms of the life expectancy of the average UK citizen, to 105,000 plex-2020. Scaling up for the UK, this comes to 438,000 deaths, equivalent to 125,000 plex-2020. The average death toll per year is thus about 88,000 across the UK, equivalent to 25,000 plex-2020.

7. Economic consequences of pursuing a policy of restriction long term

Although the scenarios considered in Section 5 may differ quantitatively in a number of their outcomes, the attenuation, $f_{\Delta R0}$, of the basic reproduction number, lies within $\sim \pm 5\%$ of the value, 0.63, with the exception of the case where only 10% of the population belongs to cohort 1, in which case the optimal value is ~10% lower at 0.57.

A further common feature is that herd immunity is not bestowed on the country by the end of the second wave.

In all cases, too, the second phase of the epidemic is long drawn out. There will still be a low level of new cases by the autumn of 2021 under all scenarios and these would continue into 2022 and beyond. No general let up on restrictions is possible because the situation is unstable, and the lack of herd immunity means that a further epidemic will always be likely.

Section 6 examines the situation where there is a further loosening of restrictions on 1st August 2021 followed by a removal of all social distancing measures on 1 March 2023. While this leads to herd immunity being achieved by the end of December 2023, the disease wave continues, albeit at a steadily diminishing level, for another year before being finally vanquished.

The long time lags mean that, in the absence of a vaccine or a cure, the strategy of close restriction would not allow the country to begin clearing itself of the grip of Covid-19 until roughly three years from now (Table 4).

What is the likely effect on the economy? A simple economic model is proposed in Appendix C whereby proportionality is assumed between the fractional change in weekly national output and the easing efficiency. This is matched to the OBR figures for the 2nd quarter of 2020, during which quarterly GDP was forecast to drop by 35% if the lockdown lasted 3 months.

Applying the lockdown economic model developed to the Extended Base Case detailed in Section 6 predicts a 23.5% fall in annual GDP in 2020, followed by a further drop in 2021. See Figure 5.

Recovery begins in 2022, but from a very low base. Economic output does not return to its 2019 level until 2024, and the average annual GDP in the 5 years, 2020 to 2024 inclusive, is 13% down on its 2019 value. See Figure 6.

Under the policy of restriction outlined above, the coronavirus slump would be substantially more severe than that associated with the financial crash of 2007 – 2009 and it would have a similar duration.

While it might be argued that human ingenuity might generate possibilities for additional growth during those 5 years, such a beneficial effect would be balanced and possibly outweighed by the likelihood that very many firms would go out of business. Very high levels of public debt as a result of measures to provide partial protection to industries during the lockdown would have a further depressing effect on the economy.

Applying the J-value-based Bristol curve[1], the change in population-average life expectancy, ΔX, is related to change in GDP per head, ΔG, by:

$$\frac{X_0 - \Delta X}{X_0} = \left(\frac{G_0 - \Delta G}{G_0}\right)^{1-\varepsilon} \qquad (11)$$

where X_0 is the population-average life expectancy, G_0 is the GDP per head and ε is risk-aversion, found to take the value, 0.91, for a developed country such as the UK. Hence

$$\Delta X = X_0\left(1-\left(\frac{G_0-\Delta G}{G_0}\right)^{1-\varepsilon}\right) \qquad (12)$$

Applying equation (12), using the average figure of a 13% reduction in GDP per head over the 5 years, 2020 to 2024, suggests the associated decrease in population-average life expectancy, ΔX, will be 0.52 years. This implies that the UK's new impoverishment will cause it to lose 67,000,000 x 0.52 = 34.9 million years of life expectancy.

The equivalent loss in terms of the life expectancy of the average UK citizen is 830,000 plex-2020. This lies midway between the UK's losses[2] in WWI and WWII.

Option 0 or "business as usual" detailed in the previous study[2] listed the life expectancy lost to the coronavirus as 16,688,465 life years. Correcting this for victims actually losing 12 years on average, rather than 16.7, leads this figure to reduce to 11,780,093 years, equivalent to 280,000 plex-2020. The loss of life from more vulnerable people contracting the coronavirus over the 5 year period in the Extended Base Case is lower at 125,000 plex-2020. This represents a saving of 155,000 plex-2020 over the worst-case scenario of Option 0.

However, this still leaves a final deficit of life lost of (830,000 – 155,000) = 675,000 plex-2020 as a result of the economic damage associated with the restriction strategy. This figure, brought about by the policy of restriction and gradual reduction, is greater than the UK's total loss of 525,000 plex-2020 during the six years of the Second World War.

8. Discussion

The epidemic of Covid-19 is a classic positive feedback process, and, as such, very difficult to control. Increased cases lead to an even larger number in the next generation and then a multitude more in the generation after that. Eventually the coronavirus starts running out of people left to infect and the outbreak recedes. But the majority of people are likely to have had the disease by the time the epidemic is over.

Lockdowns work by cutting the number of people who need to be immune from the normally required level, roughly 60% in the absence of lockdown restrictions, temporarily down to zero. But a country will only be fully safe once three fifths of its people have become resistant to infection, whether it is from past illness or by inoculation.

The ideal, in the absence of a vaccine, would be to increase slowly the number of people infected with Covid-19 until the level where herd immunity was reached, thus allowing an unstressed NHS to cope with a steady flow of serious cases. But positive feedback means that Covid-19 is difficult to keep in check. The Extended Base Case presented in Section 6 illustrates how it may be possible to keep the number of daily new cases each day down to a relatively low level, although even then the peak is about four times the highest level seen in early April 2020.

The positive feedback nature of the epidemic means that margins are inevitably tight before control is lost. The UK public has performed miraculously well in holding to a regime of strict lockdown for the past month and has thus given the NHS time to recover, regroup and extend its capacity. But maintaining 80% of the effectiveness of the full lockdown for the next 15 months and then about 50% of its effectiveness for a further 18 months to 1 March 2023 looks a difficult exercise for the British people.

Success in the task would mean that safety from the virus would eventually be achieved, but it would take until 2024, perhaps late into 2024, before that goal was finally attained.

Even if the nation were able to keep to a plan rather like that outlined in Section 6, the damage to the economy could be expected to be very large and to lead to a very substantial net loss in terms of human life, as shown in Section 7. The remedy would have caused more life to be lost than it restored.

It might be part of the Government's plan that a vaccine should become available. Certainly it is a hope. But this beneficial outcome is, of course, not certain. In any case, it is difficult to see the necessary development, mass manufacture and roll out to 40 million UK citizens taking less than a year or 18 months even under optimistic assumptions. In any case it is likely that substantial damage to the economy and hence the nation's health would result from a policy of waiting for a vaccine, even if that were to arrive within 18 months, because of the intimate relationship between GDP per head and national life expectancy.

An additional hope might be that a beefed up regime of contact tracing could shoulder part of the burden currently carried by social distancing.

This study has focused on how the main strategy in use by Government, namely lockdown followed by a staged lifting of restrictions, could proceed into the future. A common feature to all the scenarios looked at is that the timescales are long, with the epidemic liable to play out over many years in each case. In the Extended Base Case, five years will have passed before final resolution of the crisis under the policy of gradually relaxing restrictions.

There must be questions about the practicality of maintaining a substantial fraction of the effectiveness of the lockdown over such a lengthy period, much longer than has been achieved so far.

9. Limitations of the data and of the model

One significant uncertainty is the mortality rate amongst the population as a whole after contracting Covid-19. The baseline figure used in this research is 0.99%. However, a study based on testing for both virus and antibodies conducted on 1,000 people in the German municipality of Gangelt, near the border with the Netherlands, suggested that it should be 0.37%.[20] This would reduce the number of deaths calculated from SARS-CoV-2 by a factor of almost 3. Such a revision would not, however, affect the calculation of the amount of life lost as a result of national economic impoverishment. The case against the policy of lockdown followed by gradual relaxation would be strengthened if the German study is borne out.

The models have been optimised to data on new infections. While such a model can give a reasonable indication of the number of people who will be admitted to hospital, it will be less precise in its calculations relating to how long patients will need to remain in hospital.

There is no one set of variables that characterises the reported daily new cases of Covid-19 uniquely even when the number of states is as low as four, as in this model. All the parameter sets listed in Tables 1, 2, 3 and 5 match the transient data equally well. This militates against precise quantitative predictions of the effect of the epidemic, at least at present. On the other hand, the modes of the epidemic's dynamic behaviour are likely to have been captured.

The long duration associated with the epidemic's waves is a feature common to all the scenarios. All the disease transients tend to be long-drawn out, with tails resulting from an initial easing of restrictions in May 2020 lasting to the end of 2021 or later. This generic feature has a significant bearing on the likely length and severity of the coronavirus recession, which it is the principal purpose of this paper to explore.

10. Conclusions

The paper has used the J-value method to cast light on the policy of lockdown and its relaxation by linking findings from epidemiological and economic analysis.

While considerable uncertainties remain with the epidemiology, there is no doubt that the positive feedback nature of the Covid-19 epidemic will make controlling the present outbreak a very difficult task, especially when the main regulating tool is the co-ordinated behaviour of 67 million people.

The likely economic effect has been analysed of the years-long process to move out of lockdown sufficiently slowly so as not to cause excessive strain on the health services.

[20] Streeck, H., Hartmann, G., Exner, M. and Schmid, M., 2020, Vorläufiges Ergebnis und Schlussfolgerungen der COVID-19 Case-Cluster-Study (Gemeinde Gangelt), April 9, https://www.land.nrw/sites/default/files/asset/document/zwischenergebnis_covid19_case_study_gangelt_0.pdf

The Extended Base Case suggests that annual GDP will fall by 23.5% in 2020 and will not recover to pre-lockdown levels until 2024.

The coronavirus recession is likely to be significantly deeper than that associated with the 2007 – 2009 financial crash and of roughly equal duration if a strategy of lockdown followed by a very gradual easing of restrictions is pursued conscientiously.

The predictable link between GDP per head and population-average life expectancy suggests that, whatever its immediate attractions, the policy of restriction followed by gradual relaxation might, if adopted, become responsible for a net cost in terms of average human lives comparable to the UK's sacrifice over the six years of World War Two.

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Appendix A. Multi-cohort model

Of the total, *N*, of people in the national population it is assumed that a number, N_i, will respond in a certain way after contracting the disease, while other groups will react in different ways. The basic reproduction number, R_{0i}, and the average time between successive generations, $\tau_{\text{inf},i}$, will then be representative of cohort, *i*.

A schematic of the process of infection and recovery is given in Figure 7.

It is assumed that no-one will have been exposed to the infectious agent at the beginning of the outbreak, time $t = 0$, and so the number, $n_{s,i}$, of susceptible people in cohort, *i*, will be

$$n_{s,i}(0) = N_i \qquad i = 1, 2, ..., m \qquad \text{(A.1)}$$

where *m* is the number of cohorts.

Let $n_i(t)$ be the number of people who are infectious with the coronavirus in the i^{th} cohort at time, *t*, and let $n(t)$ be the total number of infectious people, so that

$$n(t) = \sum_{i=1}^{m} n_i(t) \qquad \text{(A.2)}$$

If the fraction of the population in cohort, *i*, is θ_i, then the number of people in cohort *i* before the disease has emerged will be $N_i = \theta_i N$. Let $n_{r,i}$ be the number of people in cohort, *i*, who were infected previously but who have now recovered or died. The number of still susceptible people, $n_{s,i}$, in cohort, *i*, at time, *t*, will then be

$$n_{s,i}(t) = N_i - n_i(t) - n_{r,i}(t) \qquad i = 1, 2, ..., m \qquad \text{(A.3)}$$

The probability will be $n_{s,i}(t)/N$ that an infection event at time, t, will involve susceptible people from cohort, *i*. The rate, $dn_{x,i}/dt$, at which people in cohort, *i*, are being infected will have contributions from infectious people from all cohorts. By analogy with Thomas's equation (A.22)[21],

$$\frac{dn_{x,i}}{dt}(t) = \frac{n_{s,i}(t)}{N} \sum_{k=1}^{m} R_{0k} \frac{n_k(t)}{\tau_{\text{inf},k}} \qquad i = 1, 2, ..., m \qquad \text{(A.4)}$$

[21] Thomas, P., 2020, "J-value assessment of how best to combat COVID-19", *Nanotechnology Perceptions*, Vol.16, pp. 16–40.

The summation in equation (A.4) allows for cross-infection between groups.

The number of people passing on their infection per day and moving to recover or die will be determined by which cohort they belong to. For cohort, *i*,

$$\frac{dn_{r,i}}{dt}(t) = \frac{n_i(t)}{\tau_{\inf,i}} \qquad i = 1,2,...,m \qquad \text{(A.6)}$$

The net rate of growth of infectious people in the i^{th} cohort can be found by subtracting equation (A.6) from equation (A.5):

$$\frac{dn_i}{dt}(t) = \frac{dn_{x,i}}{dt}(t) - \frac{dn_{r,i}}{dt}(t) \qquad i = 1,2,...,m \qquad \text{(A.7)}$$

Hence

$$\frac{dn_i}{dt}(t) = \frac{n_{s,i}(t)}{N}\sum_{k=1}^{m} R_{0k}\frac{n_k(t)}{\tau_{\inf,k}} - \frac{n_i(t)}{\tau_{\inf,i}} \qquad i = 1,2,...,m \qquad \text{(A.8)}$$

an equation which may be seen to be a generalization of equation (A.14) in Thomas[1].

Appendix B. Degeneration of the multi-cohort model to the single-cohort model

In the case where all cohorts have the same basic reproduction number and the same average time between generations: $R_{0i} = R_0$ and $\tau_{\inf,i} = \tau_{\inf}$ for all $i = 1,2,...,m$, then equation (A.8) degenerates to

$$\frac{dn_i}{dt}(t) = \frac{n_{s,i}(t)}{N}\frac{R_0}{\tau_{\inf}}\sum_{k=1}^{m} n_k(t) - \frac{n_i(t)}{\tau_{\inf}} \qquad i = 1,2,...,m \qquad \text{(B.1)}$$

Summing over all $i : i = 1,2,...,m$ gives:

$$\sum_{i=1}^{m}\frac{dn_i}{dt}(t) = \frac{R_0}{\tau_{\inf}}\sum_{k=1}^{m} n_k(t)\frac{1}{N}\sum_{i=1}^{m} n_{s,i}(t) - \frac{1}{\tau_{\inf}}\sum_{i=1}^{m} n_i(t) \qquad \text{(B.2)}$$

But the sum of the number of susceptible people across all groups is the sum total, n_s :

$$n_s(t) = \sum_{i=1}^{m} n_{s,i}(t) \qquad \text{(B.3)}$$

Meanwhile, by equation (A.2), $n(t) = \sum_{i=1}^{m} n_i(t)$ while formal differentiation of that equation gives:

$$\frac{dn}{dt}(t) = \sum_{i=1}^{m} \frac{dn_i}{dt}(t) \qquad \text{(B.4)}$$

Thus, substituting from equations (A.2), (B.3) and (B.4) into equation (B.2) produces

$$\frac{dn}{dt}(t) = \frac{n(t)}{\tau_{\text{inf}}}\left(R_0 \frac{n_s(t)}{N} - 1 \right) \qquad \text{(B.5)}$$

In an analogous way, summation of equation (A.6) over all cohorts will give:

$$\frac{dn_r}{dt}(t) = \frac{n(t)}{\tau_{\text{inf}}} \qquad i = 1, 2, ..., m \qquad \text{(B.6)}$$

Equations (B.5) and (B.6) describe the single cohort model used previously (equations (A.14) and (A.9) respectively of Thomas[1]).

Integrating equations (A.8) and (A.6) from the starting conditions:

$$\sum_{i=1}^{m} n_i(0) = n(0) \qquad \text{(B.7)}$$

and

$$\sum_{i=1}^{m} n_{r,i}(0) = n_r(0) \qquad \text{(B.8)}$$

for $R_{0i} = R_0$ and $\tau_{\text{inf},i} = \tau_{\text{inf}}$ for all $i = 1, 2, ..., m$, will give the same results as integrating equations (B.5) and (B.6).

Appendix C. Level of annual GDP while restrictions are in force

Let W_A be the nation's total output (work) produced in a year, $W_Q^{(n)}$ be the amount produced in quarter *n* and $W_W^{(n,k)}$ represent the amount produced in the k^{th} week of quarter *n*, all measured in units of currency. It follows that

$$W_A = \sum_{n=1}^{4} W_Q^{(n)} \qquad \text{(C.1)}$$

and that

$$W_Q^{(n)} = \sum_{k=1}^{13} W_W^{(n,k)} \qquad \text{(C.2)}$$

The change in output in quarter *n* compared with the previous quarter may be written:

$$\begin{aligned}\Delta W_Q^{(n)} = W_Q^{(n)} - W_Q^{(n-1)} &= \sum_{k=1}^{13} W_W^{(n,k)} - \sum_{k=1}^{13} W_W^{(n-1,k)} \\ &= \sum_{k=1}^{13} W_W^{(n,k)} - 13 W_W^{(n-1)} = \sum_{k=1}^{13} \left(W_W^{(n,k)} - W_W^{(n-1)} \right)\end{aligned} \qquad \text{(C.3)}$$

which frames the difference in terms of the average output per week, $W_W^{(n-1)}$, in quarter $n-1$:

$$W_W^{(n-1)} = \frac{1}{13} \sum_{k=1}^{13} W_W^{(n-1,k)} \qquad \text{(C.4)}$$

The relative change in output from one quarter to the next, usually expressed in percentage terms, may be written $\Delta p_Q^{(n,n-1)}$, where:

$$\Delta p_Q^{(n,n-1)} = \frac{W_Q^{(n)} - W_Q^{(n-1)}}{W_Q^{(n-1)}} \qquad \text{(C.5)}$$

Hence the output in quarter *n* is given in terms of the output in the previous quarter by

$$W_Q^{(n)} = W_Q^{(n-1)} + \Delta p_Q^{(n,n-1)} W_Q^{(n-1)} = \left(1 + \Delta p_Q^{(n,n-1)}\right) W_Q^{(n-1)} \qquad \text{(C.6)}$$

Using equation (C.2), equation (C.6) may be written:

$$\sum_{k=1}^{13} W_W^{(n,k)} = \left(1 + \Delta p_Q^{(n,n-1)}\right) \sum_{k=1}^{13} W_W^{(n-1,k)} \qquad \text{(C.7)}$$

However, it would also be possible to find the left-hand side of equation (C.7) by considering the change, $\Delta p_W^{(n,n-1,k)}$ in the week's output compared with the corresponding week in the previous quarter:

$$\sum_{k=1}^{13} W_W^{(n,k)} = \sum_{k=1}^{13} \left(1 + \Delta p_W^{(n,n-1,k)}\right) W_W^{(n-1,k)} \qquad \text{(C.8)}$$

Combining equations (C.7) and (C.8) and rearranging shows that $\left(1+\Delta p_Q^{(n,n-1)}\right)$ is the weighted average value of $\left(1+\Delta p_W^{(n,n-1,k)}\right)$:

$$\left(1+\Delta p_Q^{(n,n-1)}\right) = \frac{1}{\sum_{k=1}^{13} W_W^{(n-1,k)}} \sum_{k=1}^{13} \left(1+\Delta p_W^{(n,n-1,k)}\right) W_W^{(n-1,k)} \qquad \text{(C.9)}$$

In conditions of flat growth, such as pertained in the UK in 2019 (1.1% per year[22]) and the first two months of 2020 – the 3-month rolling average growth to the end of February 2020 was just 0.1%,[23] then it is reasonable to use the approximation:

$$W_W^{(n-1,k)} \approx W_W^{(n-1)} \qquad \text{(C.10)}$$

where $W_W^{(n-1)}$ is the average value given in equation (C.4). Substituting back into equation (C.9) then gives:

$$\left(1+\Delta p_Q^{(n,n-1)}\right) \approx \frac{W_W^{(n-1)}}{13 W_W^{(n-1)}} \sum_{k=1}^{13} \left(1+\Delta p_W^{(n,n-1,k)}\right) \qquad \text{(C.11)}$$

It is clear from equation (C.11) that the relative change in quarterly output is approximately equal to the arithmetic mean of the relative change in weekly output:

$$\Delta p_Q^{(n,n-1)} \approx \frac{1}{13} \sum_{k=1}^{13} \Delta p_W^{(n,n-1,k)} \qquad \text{(C.12)}$$

It is known from the OBR report[3] that the social distancing restrictions of lockdown will exert a dominant influence on the economy's capacity to produce. To provide a stable reference point, each week's output during the time restrictions are in force will be calculated with reference to the weekly output in the first quarter of 2020, which is held to represent mainly normal conditions. In fact, since the lockdown started at the beginning of week 13 of 2020, a special average needs to be taken to cover the first 12 weeks only:

$$W_W^{(1_{2020})} = \frac{1}{12} \sum_{k=1}^{12} W_W^{(1_{2020},k)} \qquad \text{(C.13)}$$

[22] Office for National Statistics, 2020, GDP first quarterly estimate, UK: October to December 2019, February 11, https://www.ons.gov.uk/economy/grossdomesticproductgdp/bulletins/gdpfirstquarterlyestimateuk/octobertodecember2019

[23] Office for National Statistics, 2020, GDP monthly estimate, UK: February 2020, 9 April. https://www.ons.gov.uk/economy/grossdomesticproductgdp/bulletins/gdpmonthlyestimateuk/february2020

where the index, 1_{2020}, denotes the first 12 weeks of 2020. The average weekly output, $W_W^{(1_{2020})}$, in the first 12 weeks of 2020 will be take as the reference point against which changes in weekly output will be judged.

The effect of the lockdown will be modelled by assuming that the weekly fractional change in output with respect to the average week's output in the first 12 weeks of 2020, $\Delta p_W^{(n,1_{2020},k)}$, will be proportional to the easing efficiency, $\eta^{(n,k)}$, with constant of proportionality, *h*:

$$\Delta p_W^{(n,1_{2020},k)} = h\eta^{(n,k)} \qquad \text{(C.15)}$$

Thus the output of week *k* in quarter *n* can be expressed as

$$W_W^{(n,k)} = \left(1 + \Delta p_W^{(n,1_{2020},k)}\right) W_W^{(1_{2020})} \qquad \text{(C.16)}$$

while the total output of quarter *n* will be the sum:

$$\begin{aligned} W_Q^{(n)} &= \sum_{k=1}^{13} \left(1 + \Delta p_W^{(n,1_{2020},k)}\right) W_W^{(1_{2020})} \\ &= \sum_{k=1}^{13} \left(1 + h\eta^{(n,k)}\right) W_W^{(1_{2020})} \end{aligned} \qquad \text{(C.17)}$$

Quarterly production in the first quarter of 2020 will consist of 12 weeks unimpaired and then 1 week where the easing efficiency is 1.0. Hence

$$W_Q^{(1)} = 12W_W^{(1_{2020})} + (1-h)W_W^{(1_{2020})} = (13-h)W_W^{(1_{2020})} \qquad \text{(C.18)}$$

Under the OBR scenario the output of the second quarter of 2020 will consist of 12 weeks where $\eta = 1$ and one where restrictions are being eased over a 13 week period, so that $\eta^{(2,13)} = 1 - 1/13 = 0.92$. Hence:

$$W_Q^{(2)} = 12(1-h)W_W^{(1_{2020})} + (1-0.92h)W_W^{(1_{2020})} = (13-12.92h)W_W^{(1_{2020})} \qquad \text{(C.19)}$$

The OBR suggests that there is a fall of 35% in GDP in the second quarter. Hence

$$\Delta p_Q^{(2,1)} = \frac{W_Q^{(2)} - W_Q^{(1)}}{W_Q^{(2)}} = \frac{(13-12.92h)W_W^{(1_{2020})} - (13-h)W_W^{(1_{2020})}}{(13-h)W_W^{(1_{2020})}} = -0.35 \qquad \text{(C.20)}$$

which has the solution $h = 0.37$.

The annual output may now be determined in terms of the average weekly output in the first 12 weeks of 2020:

$$W_A = \sum_{n=1}^{4} W_Q^{(n)} = W_W^{(1_{2020})} \sum_{n=1}^{4} \sum_{k=1}^{13} \left(1 + h\eta^{(n,k)}\right) = W_W^{(1_{2020})} \sum_{m=1}^{52} \left(1 + h\eta^{(m)}\right) \qquad \text{(C.21)}$$

where the indexing has been simplified to denote the week's place in the year.

Under the Extended Base Case scenario, in 2020 there will be 12 weeks pre-lockdown where the easing efficiency is 0, 6 weeks where $\eta = 1.0$, three weeks of easing out of lockdown in the initial relaxation, with $\eta = 0.97, 0.9$ and 0.83 in sequence, and then 31 weeks where $\eta = 0.8$. (See Figure 4 for the dates of successive relaxations.)

Using these figures in equation (C.20) gives

$$W_{2020} = 39.61 W_W^{(1_{2020})} \qquad \text{(C.22)}$$

Given a growth rate of about 1% per annum, we may calculate the GDP for 2019 as:

$$W_{2019} = 52\left(1 - 0.005\right) W_W^{(1_{2020})} = 51.74 W_W^{(1_{2020})} \qquad \text{(C.23)}$$

Thus the ratio of GDP in 2020 to GDP in 2019 may be calculated as 76.5%, implying an annual fall of 23.5%.

In a similar fashion, 2021 will see 31 weeks where $\eta = 0.8$ followed by 21 weeks where $\eta = 0.47$. Hence

$$W_{2021} = 39.17 W_W^{(1_{2020})} \qquad \text{(C.24)}$$

This is 75.7% of 2019's GDP, representing a further annual fall of 1.1% from the GDP of 2020.

There will be 52 weeks at $\eta = 0.47$ in 2022, and this gives

$$W_{2022} = 42.96 W_W^{(1_{2020})} \qquad \text{(C.25)}$$

This indicates economic growth of 9.7% per year in 2022. However, the output for 2022 is still only 83% of the GDP of 2019.

In 2023 there will be 9 weeks at $\eta = 0.47$ before restrictions are removed and $\eta = 0$ for the last 43 weeks. This implies:

$$W_{2023} = 50.43 W_W^{(1_{2020})} \qquad \text{(C.26)}$$

Economic growth climbs to 17.4% per annum in 2023. The output reaches 97.5% of the GDP of 2019.

There are no restrictions in 2024, and GDP rises to 100.5% of the 2019 level. Growth is 3.1% per annum.

Figure 5 shows the percentage change in annual GDP year on year. Figure 6 shows the nation's output as a percentage of 2019's annual GDP.

The average shortfall on the GDP of 2019 over the 5 years 2020 to 2024 inclusive is 13%.

Basic reproduction number, cohort 1, R_{01}	2.90	Total positive tests to 10 April 2020	71,353 (sum of reported figures)
Average time between generations, cohort 1, $\tau_{\mathrm{inf},1}$	8.23 days	Total infected people to 10 April 2020	359,061
Basic reproduction number, cohort 2, R_{02}	2.34	Maximum daily confirmed cases in the 2nd wave	16,516
Average time between generations, cohort 2, $\tau_{\mathrm{inf},2}$	10.18 days	Maximum Covid-19 hospital in-patients during 2nd wave	60,775
Fractional decrease in the two basic reproduction numbers after lockdown, $f_{\Delta R0}$	0.6380	Total positive tests by end of the 2nd wave (Cohort 1: April 10 testing conventions)	3,477,699
Number of infectious people in cohort 2 on 30 January 2020, $n_2(0)$	13	Total population infected in England by end of outbreak	17,388,502
Total number hospitalised by the end of the 2nd wave	1,564,965	Total number of deaths by the end of the 2nd wave	172,146
Death rate for the population as a whole	0.99%	Equivalent average lives lost (plex-2020)	49,185

Table 1(i) Parameter values for Base Case, θ_1 = 0.2, η = 0.8 [Section 5.1.1 Scenario (i)]. Optimised parameters shaded.

Basic reproduction number, cohort 1, R_{01}	2.58	Total positive tests to 10 April 2020	71,353 (sum of reported figures)
Average time between generations, cohort 1, $\tau_{\text{inf},1}$	6.28 days	Total infected people to 10 April 2020	358,969
Basic reproduction number, cohort 2, R_{02}	2.41	Maximum daily confirmed cases in the 2nd wave	17,027
Average time between generations, cohort 2, $\tau_{\text{inf},2}$	10.82 days	Maximum Covid-19 hospital in-patients during 2nd wave	47,983
Fractional decrease in the two basic reproduction numbers after lockdown, $f_{\Delta R0}$	0.6338	Total positive tests by end of the 2nd wave (Cohort 1: April 10 testing conventions)	3,546,533
Number of infectious people in cohort 2 on 30 January 2020, $n_2(0)$	12	Total population infected in England by end of outbreak	17,332,670
Total number hospitalised by the end of the 2nd wave	1,595,940	Total number of deaths by the end of the 2nd wave	175,553
Death rate for the population as a whole	0.99%	Equivalent average lives lost (plex-2020)	50,158

Table 1(ii) Parameter values for Section 5.1.1 Scenario (ii), θ_1 = 0.2, η = 0.8. Optimised parameters shaded.

Basic reproduction number, cohort 1, R_{01}	3.03	Total positive tests to 10 April 2020	71,353 (sum of reported figures)
Average time between generations, cohort 1, $\tau_{\text{inf},1}$	11.42 days	Total infected people to 10 April 2020	358,792
Basic reproduction number, cohort 2, R_{02}	2.71	Maximum daily confirmed cases in the 2nd wave	26,326
Average time between generations, cohort 2, $\tau_{\text{inf},2}$	11.84 days	Maximum Covid-19 hospital in-patients during 2nd wave	133,096
Fractional decrease in the two basic reproduction numbers after lockdown, $f_{\Delta R0}$	0.6666	Total positive tests by end of the 2nd wave (Cohort 1: April 10 testing conventions)	4,633,144
Number of infectious people in cohort 2 on 30 January 2020, $n_2(0)$	13	Total population infected in England by end of outbreak	23,165,727
Total number hospitalised by the end of the 2nd wave	2,084,915	Total number of deaths by the end of the 2nd wave	229,915
Death rate for the population as a whole	0.99%	Equivalent average lives lost (plex-2020)	65,526

Table 1(iii) Parameter values for Section 5.1.1 Scenario (iii) θ_1 = 0.2, η = 0.8. Optimised parameters shaded.

Basic reproduction number, cohort 1, R_{01}	2.90	Total positive tests to 10 April 2020	71,353 (sum of reported figures)
Average time between generations, cohort 1, $\tau_{\text{inf},1}$	8.23 days	Total infected people to 10 April 2020	359,061
Basic reproduction number, cohort 2, R_{02}	2.34	Maximum daily confirmed cases in the 2nd wave	42,635
Average time between generations, cohort 2, $\tau_{\text{inf},2}$	10.18 days	Maximum Covid-19 hospital in-patients during 2nd wave	155,153
Fractional decrease in the two basic reproduction numbers after lockdown, $f_{\Delta R0}$	0.6380	Total positive tests by end of the 2nd wave (Cohort 1: April 10 testing conventions)	5,260,884
Number of infectious people in cohort 2 on 30 January 2020, $n_2(0)$	13	Total population infected in England by end of outbreak	26,304,423
Total number hospitalised by the end of the 2nd wave	2,367,398	Total number of deaths by the end of the 2nd wave	260,414
Death rate for the population as a whole	0.99%	Equivalent average lives lost (plex-2020)	74,404

Table 2(i) Parameter values for Section 5.1.2 varying easing efficiency, Scenario (i.), $\theta_1 = 0.2$, $\eta = 0.7$. Optimised parameters shaded.

Basic reproduction number, cohort 1, R_{01}	2.90	Total positive tests to 10 April 2020	71,353 (sum of reported figures)
Average time between generations, cohort 1, $\tau_{\text{inf},1}$	8.23 days	Total infected people to 10 April 2020	359,061
Basic reproduction number, cohort 2, R_{02}	2.34	Maximum daily confirmed cases in the 2nd wave	2,553
Average time between generations, cohort 2, $\tau_{\text{inf},2}$	10.18 days	Maximum Covid-19 hospital in-patients during 2nd wave	9,442
Fractional decrease in the two basic reproduction numbers after lockdown, $f_{\Delta R0}$	0.6380	Total positive tests by end of the 2nd wave (Cohort 1: April 10 testing conventions)	1,131,339
Number of infectious people in cohort 2 on 30 January 2020, $n_2(0)$	13	Total population infected in England by end of 2nd wave	5,656,701
Total number hospitalised by the end of the 2nd wave	509,102	Total number of deaths by the end of the 2nd wave	56,001
Death rate for the population as a whole	0.99%	Equivalent average lives lost (plex-2020)	18,000

Table 2(ii) Parameter values for Section 5.1.2 varying easing efficiency, Scenario (ii) $\theta_1 = 0.2$, $\eta = 0.9$. Optimised parameters shaded.

Basic reproduction number, cohort 1, R_{01}	2.50	Total positive tests to 10 April 2020	71,353 (sum of reported figures)
Average time between generations, cohort 1, $\tau_{\mathrm{inf},1}$	6.45 days	Total infected people to 10 April 2020	716,894
Basic reproduction number, cohort 2, R_{02}	1.91	Maximum daily confirmed cases in the 2nd wave	2,059
Average time between generations, cohort 2, $\tau_{\mathrm{inf},2}$	6.44 days	Maximum Covid-19 hospital in-patients during 2nd wave	5,966
Fractional decrease in the two basic reproduction numbers after lockdown, $f_{\Delta R0}$	0.5725	Total positive tests by end of the 2nd wave (Cohort 1: April 10 testing conventions)	713,162
Number of infectious people in cohort 2 on 30 January 2020, $n_2(0)$	20	Total population infected in England by end of 2nd wave	7,131,626
Total number hospitalised by the end of the 2nd wave	320,923	Total number of deaths by the end of the 2nd wave	35,302
Death rate for the population as a whole	0.49%	Equivalent average lives lost (plex-2020)	10,086

Table 3(i) Parameter values for Section 5.1.3, varying population fraction in Cohort 1, Scenario (i) , η = 0.8, θ_1 = 0.1. Optimised parameters shaded.

Basic reproduction number, cohort 1, R_{01}	3.89	Total positive tests to 10 April 2020	71,353 (sum of reported figures)
Average time between generations, cohort 1, $\tau_{\inf,1}$	9.36 days	Total infected people to 10 April 2020	239,296
Basic reproduction number, cohort 2, R_{02}	1.79	Maximum daily confirmed cases in the 2nd wave	22,848
Average time between generations, cohort 2, $\tau_{\inf,2}$	9.63 days	Maximum Covid-19 hospital in-patients during 2nd wave	95,540
Fractional decrease in the two basic reproduction numbers after lockdown, $f_{\Delta R0}$	0.6381	Total positive tests by end of the 2nd wave (Cohort 1: April 10 testing conventions)	4,968,418
Number of infectious people in cohort 2 on 30 January 2020, $n_2(0)$	9	Total population infected in England by end of 2nd wave	16,561,398
Total number hospitalised by the end of the 2nd wave	2,235,788	Total number of deaths by the end of the 2nd wave	245,937
Death rate for the population as a whole	1.48%	Equivalent average lives lost (plex-2020)	70,268

Table 3(ii) Parameter values for Section 5.1.3, varying population fraction in Cohort 1, Scenario (ii) , η = 0.8, θ_1 = 0.3. Optimised parameters shaded.

Date	Description	Easing efficiency
00.01 hours March 24 2020	Start lockdown	1.0
00.01 hours May 5 2020	Start of relaxation	0.8 (after 3 weeks)
00.01 hours August 1 2021	2nd relaxation	0.47
00.01 hours March 1 2023	3rd relaxation	0

Table 4. Dates of changes to restrictions, Extended Base Case scenario (Section 6)

Basic reproduction number, cohort 1, R_{01}	2.25	Total positive tests to 10 April 2020	71,353 (sum of reported figures)
Average time between generations, cohort 1, $\tau_{\inf,1}$	8.33 days	Total infected people to 10 April 2020	358,973
Basic reproduction number, cohort 2, R_{02}	2.25	Maximum daily confirmed cases in the 2nd wave	10,134
Average time between generations, cohort 2, $\tau_{\inf,2}$	8.33 days	Maximum Covid-19 hospital in-patients during 2nd wave	37,843
Fractional decrease in the two basic reproduction numbers after lockdown, $f_{\Delta R0}$	0.6170	Total positive tests by end of the 2nd wave (Cohort 1: April 10 testing conventions)	2,587,447
Number of infectious people in cohort 2 on 30 January 2020, $n_2(0)$	11	Total population infected in England by end of 2nd wave	12,937,241
Total number hospitalised by the end of the 2nd wave	1,164,351	Total number of deaths by the end of the 2nd wave	128,079
Death rate for the population as a whole	0.99%	Equivalent average lives lost (plex-2020)	36,594

Table 5(i) Parameter values for Section 5.2, matching parameters in Cohorts 1 & 2 Scenario (i) , η = 0.8, θ_1 = 0.2. Optimised parameters shaded.

Basic reproduction number, cohort 1, R_{01}	2.77	Total positive tests to 10 April 2020	71,353 (sum of reported figures)
Average time between generations, cohort 1, $\tau_{\text{inf},1}$	11.78 days	Total infected people to 10 April 2020	358,825
Basic reproduction number, cohort 2, R_{02}	2.77	Maximum daily confirmed cases in the 2nd wave	26,526
Average time between generations, cohort 2, $\tau_{\text{inf},2}$	11.78 days	Maximum Covid-19 hospital in-patients during 2nd wave	138,194
Fractional decrease in the two basic reproduction numbers after lockdown, $f_{\Delta R0}$	0.6665	Total positive tests by end of the 2nd wave (Cohort 1: April 10 testing conventions)	4,653,197
Number of infectious people in cohort 2 on 30 January 2020, $n_2(0)$	13	Total population infected in England by end of 2nd wave	23,265,992
Total number hospitalised by the end of the 2nd wave	2,093,939	Total number of deaths by the end of the 2nd wave	230,333
Death rate for the population as a whole	0.99%	Equivalent average lives lost (plex-2020)	65,810

Table 5(ii) Parameter values for Section 5.2, matching parameters in Cohorts 1 & 2 η = 0.8, θ_1 = 0.2. Optimised parameters shaded.

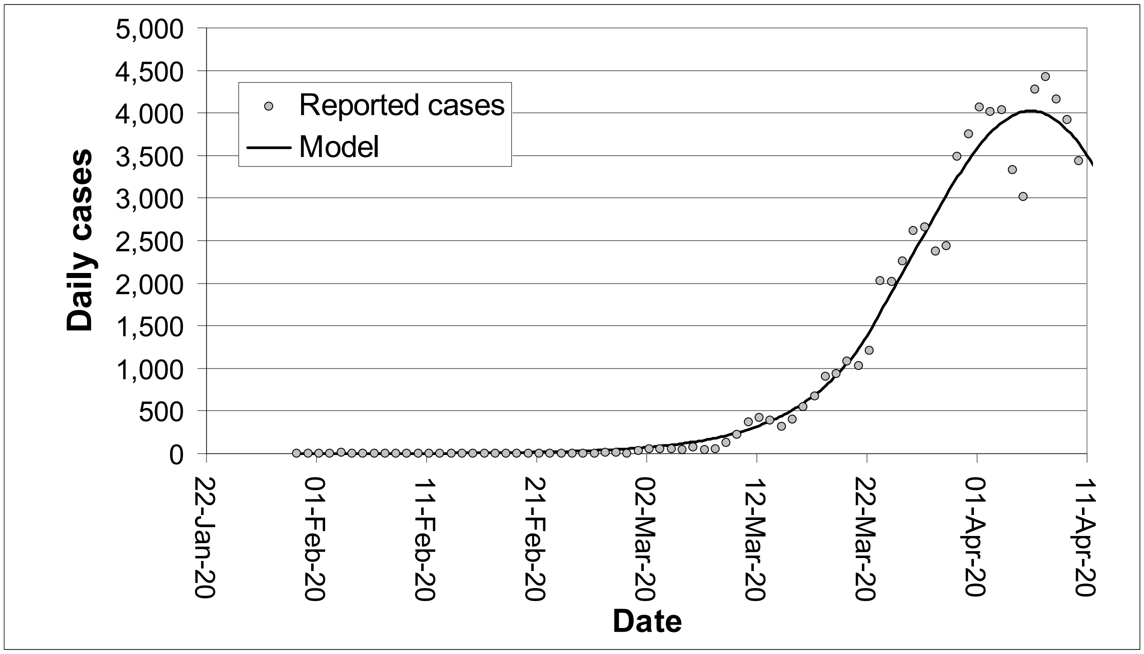

Figure 1. Matching the Base Case, $\theta_1 = 0.2$, $\eta = 0.8$ [Section 5.1.1, Scenario (i)], to the reported daily confirmed cases to 10 April 2020

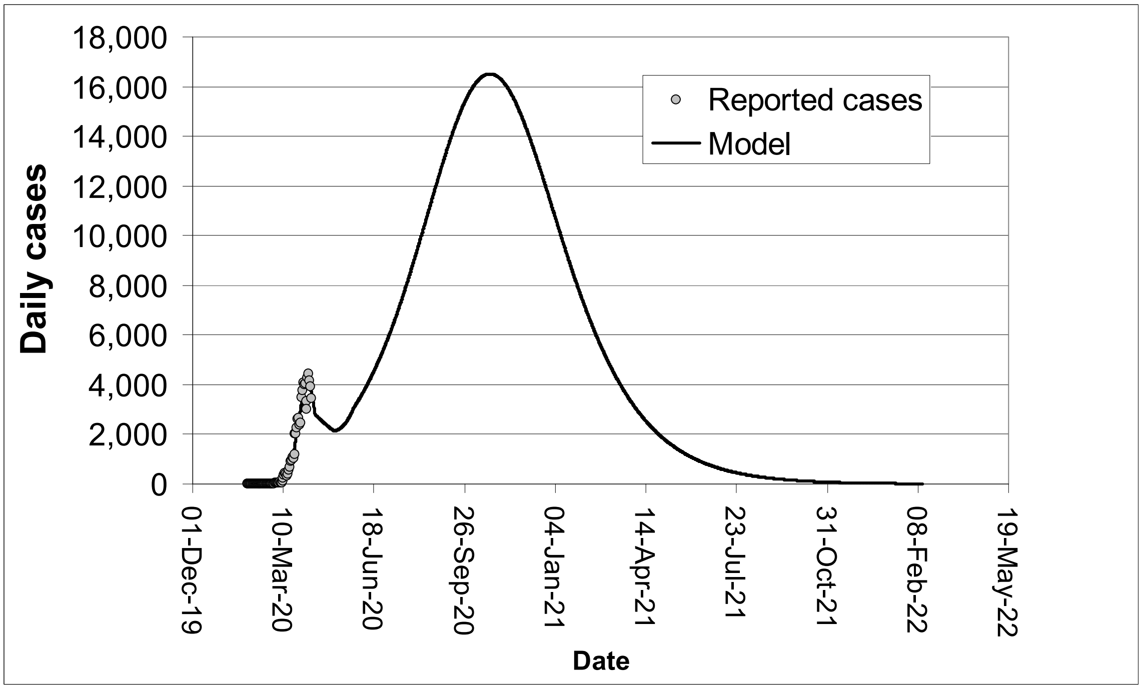

Figure 2. Base Case, $\theta_1 = 0.2$, $\eta = 0.8$ [Section 5.1.1, Scenario (i)]. Predicted number of daily cases diagnosed under the conventions existing to 10 April 2020

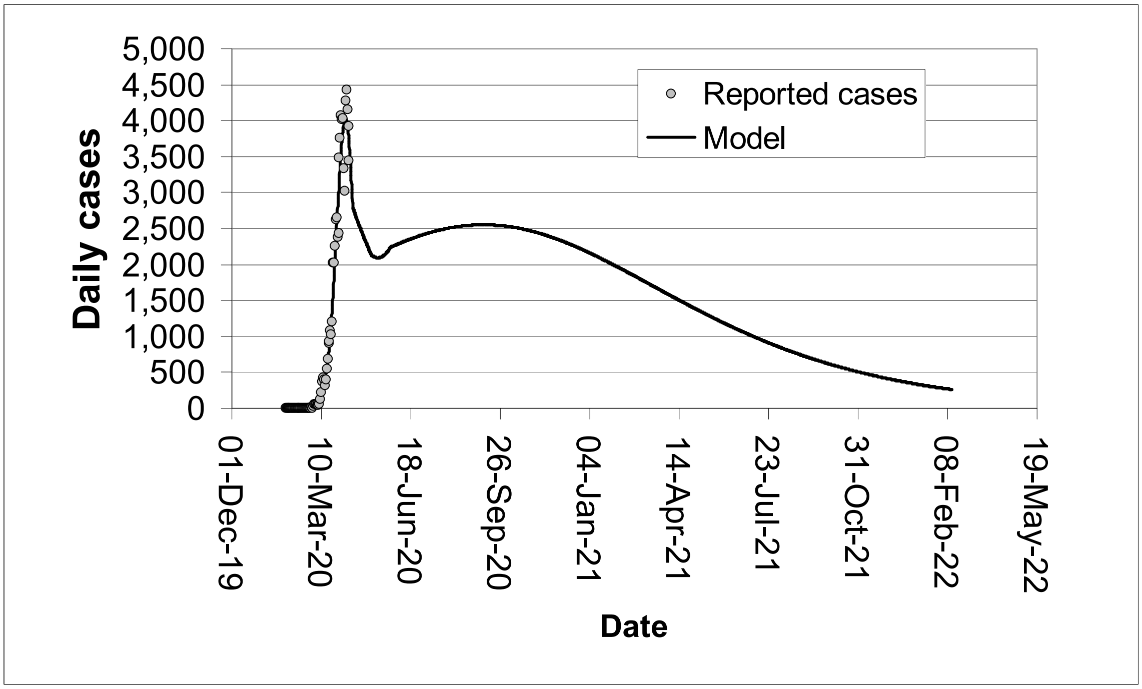

Figure 3. Section 5.1.2 Scenario ii, $\theta_1 = 0.2$, $\eta = 0.9$. Predicted number of daily cases confirmed under the testing conventions to 10 April 2020

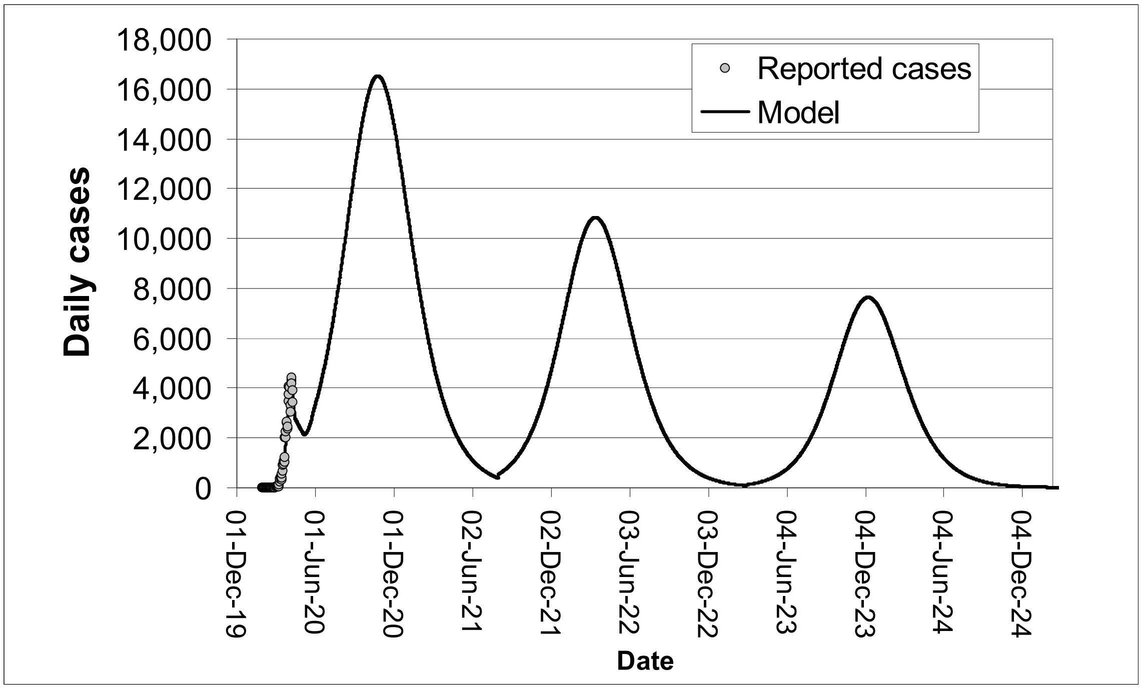

Figure 4. Extended Base Case. Daily cases [under 10 April 2020 testing conventions] when the restrictions are eased a second time on 1 August 2021 and finally abolished on 1 March 2023

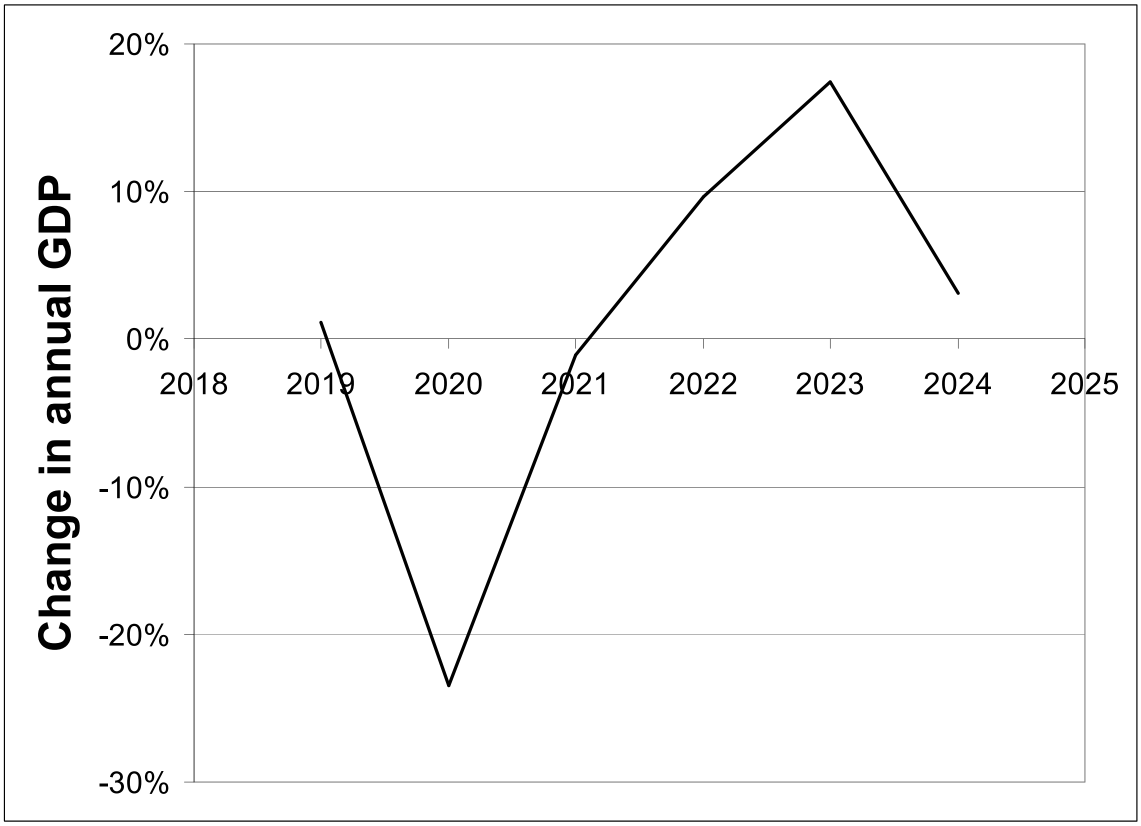

Figure 5. Change in annual GDP versus date for the Extended Base Case of Section 6.

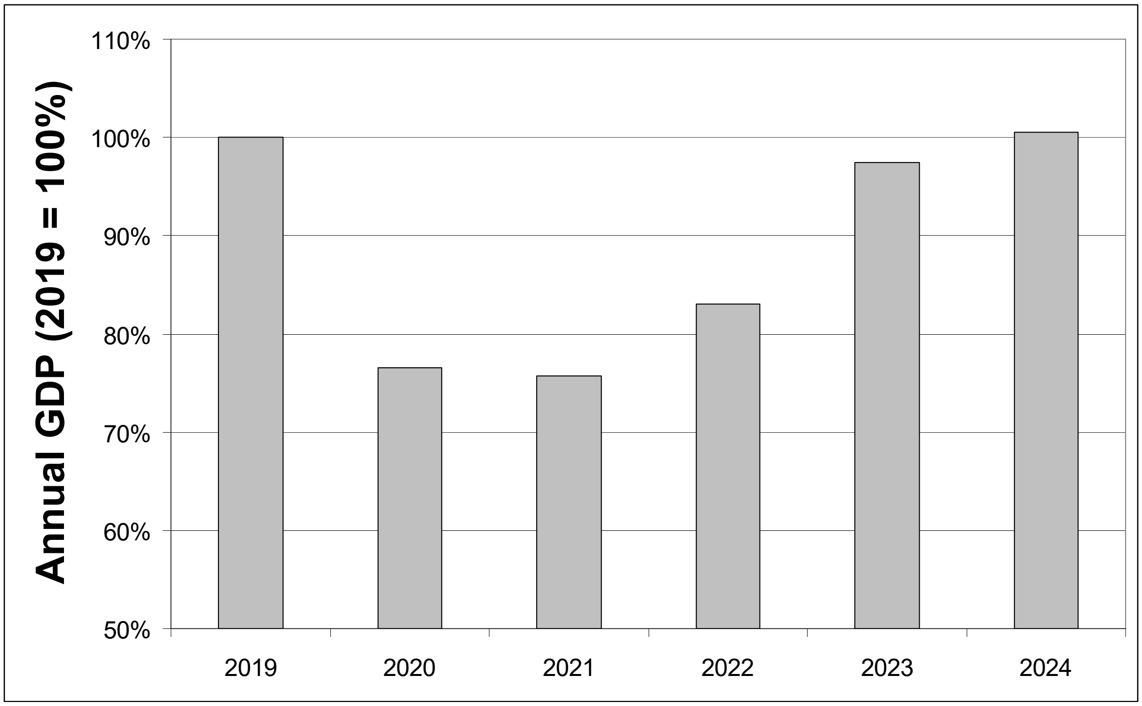

Figure 6. Annual GDP as a percentage of the figure for 2019

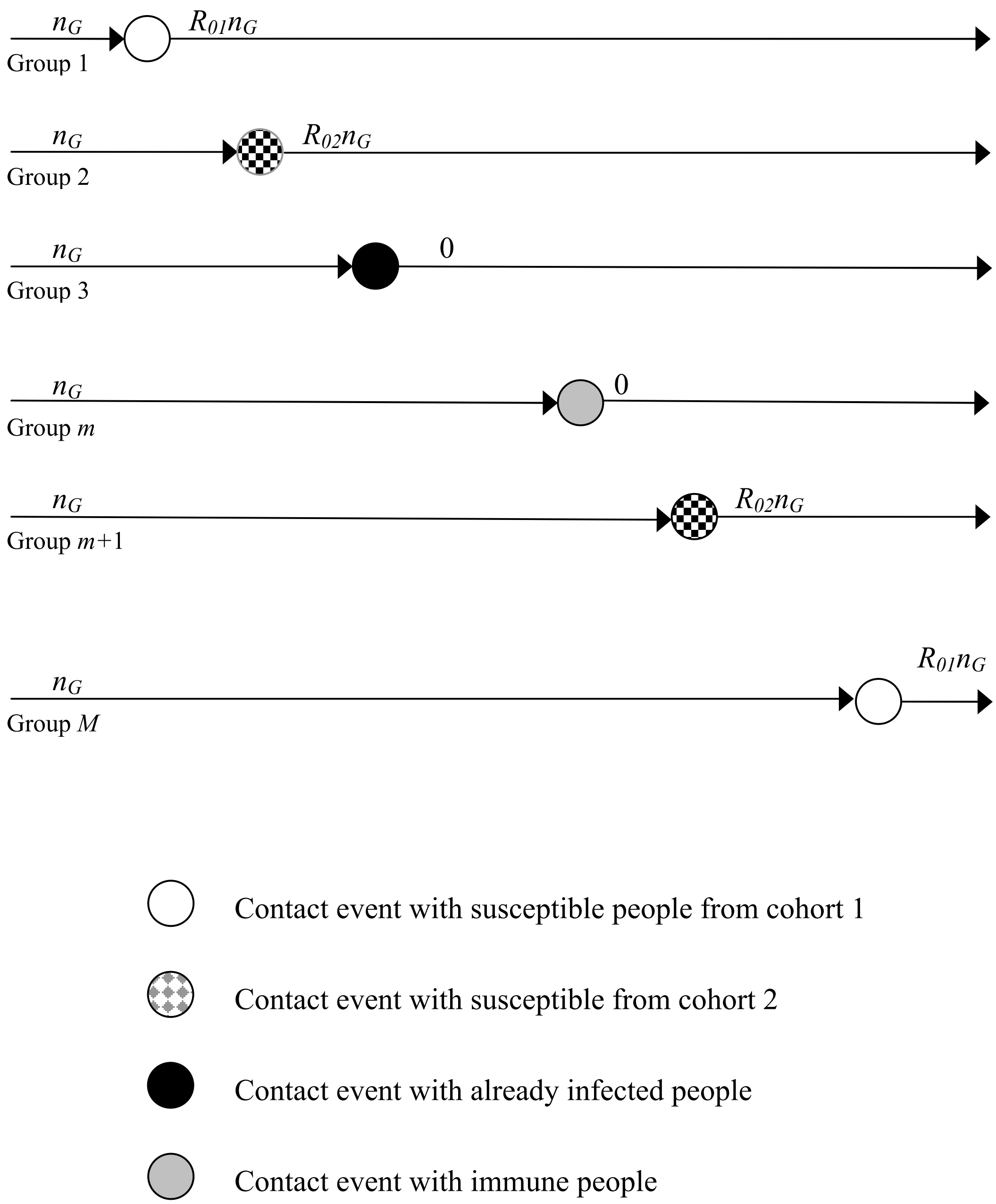

Figure 7. Schematic of the process of infection and recovery/death for 2-cohort model