Paper for *Nanotechnology Perceptions*
Submitted manuscript

Vaccine-mediated exit strategies from England's Covid-19 lockdown

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Abstract

An assessment is made of vaccine-mediated exit strategies from England's Covid-19 lockdown. Two linked methods are considered. The first, termed the gradualist approach, assumes that the R-rate is controlled to a value close to 1 and that the Social Distancing Index (SDI), which measures the nation's level of interpersonal contact, is allowed to increase continuously to the point where it can increase no further and society and the economy are fully open. The second method, termed the two-step strategy, adopts the gradualist approach initially, but then, after a certain time, the SDI is stepped to its maximum value, which implies the immediate removal of all restrictions.

It is found that, while vaccination-generated immunity makes a very valuable contribution to overall immunity, the other components, prior T-cell immunity and immunity generated by infection, are at least as important. Infection generated immunity needs to be the largest component if all restrictions are to be removed under both strategies. Exiting the lockdown to the point where all restrictions can be eased fully requires a narrow path to be followed during the spring and early summer of 2021, keeping the R-rate in a central band around 1.0. Close control of the R-rate is needed and this will require the R-rate to be measured accurately, continuously and rapidly. The gradualist approach might allow all restrictions to be lifted by the end of the summer, while the two step strategy might offer the prospect of full derestriction by the end of May 2021.

Key words: Coronavirus; Covid-19; Predictor-Corrector Coronavirus Filter; PCCF; Lockdown; Lockdown exit strategies

1. Introduction

The UK government's Vaccine Task Force was successful in securing vaccines from the first companies to gain licences for their new products. 40 million doses were ordered from the early leader, Pfizer-BioNtech, which is supplying an RNA vaccine, while 100 million are on order from the Oxford-AstraZeneca consortium, whose licensed product employs a genetically modified virus and is known as a viral vector. Orders have also been placed for17 million doses of a newly licensed vaccine from Moderna, which is again based on RNA technology.

Supplies have also been purchased of two further vaccines that are well advanced towards licensing in the UK: 60 million doses from Novomax (protein based) and 30 million of the viral-vector type from Janssen. The first four require first an initial and then a booster dose, but the Janssen vaccine is administered in one injection.

Conditioned by the requirement to ensure the vaccines progressed through all stages of development in record time (less than a year, in the event), the interval between doses explored in the field for some of the vaccines was set at three weeks. A longer period has been found usually to be desirable between injections, and the British government has decided on a space of 12 weeks to allow the fastest possible roll-out to its citizens of the first protective dose.

Subsequent research has suggested that the AstraZeneca vaccine has an average effectiveness of 76% in eliminating symptomatic infection for the period from 3 weeks to 15 weeks from the first dose and of 67% in eliminating all infections, whether symptomatic and asymptomatic[1], in the same time period. The latter figure is of key importance because it means that no transmission will occur following two thirds of potentially infectious encounters. This figure will be used in the analysis that follows, which applies to England. England constitutes a separate health jurisdiction from those of Wales, Scotland and Northern Ireland, although vaccines are distributed to each of the constituent countries of the UK on an egalitarian basis, in proportion to their populations.

It was also reported that the only cases found in the Stage 3, field trial were mild, with no serious cases and no deaths. It will be assumed, therefore, that vaccines will prevent death with an effectiveness of 95%. This figure will be assumed to apply to all vaccines used in England.

No data were supplied on the performance of the AstraZeneca vaccine in the first 3 weeks after administering the first injection. A conservative characterisation would assume that there was no benefit until three weeks after vaccination. This could be modelled by employing a step function with the value 0 up to and including 21 days and then rising asymptotically to 0.67 over the period from 22 days to 90 days. In fact it is almost certain that there will be some benefit from the vaccination earlier on, and so the process was modelled as a first-order exponential lag giving the same average value over the 13 week period. See Figure 1.

It is then assumed that the second dose of vaccine will lock in the 67% chance of preventing infection for the duration of the pandemic.

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[1] Voisey, M., et al., 2021, Single dose administration, and the influence of the timing of the booster dose on immunogenicity and effectiveness of ChAdOx1 nCoV-19 (AZD1222) vaccine, paper submitted to The Lancet, 2 February, https://papers.ssrn.com/sol3/papers.cfm?abstract_id=3777268

The time table for the Covid-19 vaccinations was based on vulnerability, interpreted principally in terms of age, although morbidities were also taken into account. For this analysis the programme of first doses in England is summarised in Figure 2. The programme is roughly sequential, with those of age 80 and over being invited first, then those between 70 and 79 and so on. Recorded daily vaccinations are used up to 6 February 2021 and then a constant 400,000 first doses a day is assumed, a rate that was exceeded on several occasions prior to 6 February 2021.

Second doses will need to be given after a maximum of twelve weeks, and this is assumed to cut the vaccination capability available for further first doses. In this analysis it is assumed that the administering of second doses begins in earnest in the second half of February 2021, and that only 80,000 first doses per day are given between the middle of February and the middle of April, with 160,000 first doses given daily from 23 April onwards.

The Office of National Statistics (ONS) has found that 92% of the adult population of Great Britain have received or would be likely to accept the COVID-19 vaccine if offered[2], and this figure has been adopted for vaccine coverage in this study.
The vaccination exercise will be complete by the beginning of September 2021.

As of late January 2021, England was living under a strictly observed lockdown, with the R-rate, the average number of people a person with Covid-19 will infect between contracting the virus and recovering, having a value of roughly 0.6, based on measurements made by the Predictor-Corrector Coronavirus Filter (PCCF)[3, 4]. This paper will consider two sets of scenarios for leaving lockdown and moving towards getting back fully to normal. The first set will involve a gradualist approach, where lockdown restrictions will be eased continuously until society and the economy are fully open. The second batch uses a two-step method, whereby a gradualist approach is used first, until the danger has been significantly mitigated by vaccination, but then all restrictions are lifted on a specified date.

The paper is laid out as follows. Section 2 of the paper derives a value for the maximum of the Social Distancing Index, which will be associated with a fully open society and economy. Section 3 explains the two basic strategies examined in the paper and the scenarios to be considered within them. A sensitivity study to be carried out on the maximum value, Σ_0, of the Social Distancing Index corresponding to full derestriction is outlined.

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[2] Office of National Statistics (ONS), 2021, Coronavirus and the social impacts on Great Britain: 12 February 2021.
https://www.ons.gov.uk/peoplepopulationandcommunity/healthandsocialcare/healthandwellbeing/bulletins/coronavirusandthesocialimpactsongreatbritain/12february2021#attitudes-to-covid-19-vaccination
[3] Thomas, P., 2021, "The options for the UK leaving the coronavirus lockdown of 2020", *Nanotechnology Perceptions*, IN PRESS
[4] *The Spectator* Covid-19 data tracker, where PCCF measurements of the number of active cases in England and England's R-rate are updated daily. https://data.spectator.co.uk/city/national

Section 4 contains the results found for the 3 scenarios analysed under the gradualist approach. Section 5 provides the results for two scenarios enacted under the potentially quicker two-step strategy. Section 6 contains sensitivity studies for the two-step strategy.

The discussion is given in Section 7 , while a note on the accuracy of the modelling is provided in Section 8. Section 9 gives conclusions.

There are 5 appendices. Appendix A explains the adaptation of the PCCF model to allow it to cater for 8 age groups. Appendix B details the modelling of the age prioritisation used in the vaccination campaign. Appendix C derives a set of base death probabilities for the different age bands, valid in the absence of vaccination. Appendix D derives a value for the vaccination maturity time constant. Appendix E offers an additional method for allowing for differences in the number of swab tests administered each day, relevant to the corrected data points provided for cases by date recorded.

2. The SDI when there are no social distancing restrictions

The maximum value of the SDI, Σ_0 , will depend on the disease involved. It is estimated to be as high as 3 for the original form of Covid-19, but the new UK variant, B.1.1.7, may be 50% more infectious[5], implying an Σ_0 -value of 4.5 for that variant. Weekly ONS infection surveys[6] covering the period between 13 January 2021 and 3 February found the prevalence of the B.1.1.7 variant in England was stable at about 60%. Taking a weighted mean of the Σ_0 -values for the original Covid-19 and the new UK variant produces an overall figure of $\Sigma_0 = 3.9$.

Given the higher infection fatality rate for the new variant, the 60:40 split between new variant and original virus would suggest a combined infection fatality rate that was 18% larger than for the original virus.

2.1. The SDI when there are no social distancing restrictions: sensitivity studies

In the sensitivity studies the case was considered where the new variant, B.1.1.7, became fully dominant in the UK, and moreover, was twice as infectious as the original virus rather than 50% more so. This has the effect of raising the derestricted SDI from 3.9 to $\Sigma_0 = 6$.

The higher death rate associated with the new variant means that the infection fatality rate is increased by a further 10%.

[5] Public Health England, 2021, What do we know about the new COVID-19 variants?, 5 February. https://publichealthmatters.blog.gov.uk/2021/02/05/what-do-we-know-about-the-new-covid-19-variants/

[6] Office of National Statistics (ONS), 2021, Coronavirus (COVID-19) Infection Survey, UK: 12 February 2021, and previous 3 issues. See https://www.ons.gov.uk/peoplepopulationandcommunity/healthandsocialcare/conditionsanddiseases/bulletins/coronaviruscovid19infectionsurveypilot/12february2021

3. The options considered

3.1 Gradualist approach

The R-rate is assigned a set point at the end of January 2021, at which is maintained thereafter by judicious control of the amount of social distancing that is allowed.

Three scenarios are explored, with the R-rate kept at set points of

1. 0.6
2. 0.95
3. 1.1

from the end of January 2021 onwards. Active infections will decrease in the first two scenarios, when $R < 1$ and will start to increase when $R > 1$ in the third.

The SDI, $\Sigma(t)$ (also known as the basic reproduction number, $R_0(t)$) is defined as the number of contacts the average person infected with Covid-19 makes with other people in the interval between becoming infected and recovering, provided that the encounter is close enough to infect the other person and would, in fact, do so in a fully "naive" population, in which no-one had any immunity.

For any given nation, Σ, will depend solely on the interpersonal behaviour of its people, which is what makes it a useful general measure of the degree of social distancing present in the nation. This behaviour will change both with the season and in response to government orders and guidance, which will change over time. Therefore the SDI must be a function of time: $\Sigma = \Sigma(t)$. A high degree of social distancing, which will happen when personal contact is minimised, will be associated with a low value of the SDI. On the other hand, a low degree of social distancing (lots of direct interaction) will lead to a high value of the SDI.

The Social Distancing Index when no restrictions on social distancing are imposed by the government or adopted voluntarily by the public will be given the name, Σ_0, and this may be considered independent of time and dependent solely on the disease in question.

The SDI, $\Sigma(t)$, may be calculated by dividing the R-rate by the fraction of the population that is still susceptible. This fraction must fall in all epidemics through the effects of both recovery from infection and vaccination, where it is offered, so that holding the R-rate constant will inevitably cause the SDI to rise.

Once the SDI reaches its highest possible value, corresponding to full opening up, the same relationship inverted implies that the R-rate can no longer be constant, but must fall instead.

3.2 Two-step strategy

3.2.1 Main cases

The two-step strategy will be applied using two initial set points for the R-rate, 0.6 and 0.95, both below 1.0, which implies that cases will start off falling.

Then full derestriction will happen at the end of May 2021, modelled by stepping the set point for the R-rate up to 1.6.

3.2.1 Sensitivity studies.

The maximum value of the SDI corresponding to full derestriction is now set at $\Sigma_0 = 6$.

The two initial set points, 0.6 and 0.95, were retained for the R-rate in the gradualist first phase. But full derestriction was delayed two months to the beginning of August 2021 in view of the ~50% increase in the Σ_0-value.

The new UK variant is thought to have a 30% higher infection fatality rate[7]. Given the move from a 60:40 split between the new UK variant and the original virus that existed in January 2021 to 100% new UK variant, the overall infection fatality rate would rise by 10% over the figure observed in January 2021.

4. Results for the gradualist approach

4.1 The R-rate is held at 0.6 from the end of January 2021 onwards

A model[8] was matched to the official data for England's "cases by date reported" from the beginning of December 2020 to the end of January 2021. An estimate was then made of the infection fatality rate over all ages that would be produced in the absence of vaccination by matching its prediction for the peak death rate in January 2021 to the highest recorded number based on specimen date. See Figure 3. The resultant figure, 1.32%, is retained for all the scenarios under the assumption that the B.1.1.7 variant: original virus prevalence remains at 60:40 throughout the rest of the epidemic. (It will be further increased by 10% to 1.45% in the sensitivity study of Section 6.)

Holding the R-rate at 0.6 from the end of January 2021 onwards produced the uniform fall in England's cases by date reported shown in Figure 4. Meanwhile the active infections in England are also predicted to fall away rapidly, dropping below 10,000 in the second half of May, as shown in Figure 5.

[7] Public Health England, 2021, What do we know about the new COVID-19 variants?, 5 February. https://publichealthmatters.blog.gov.uk/2021/02/05/what-do-we-know-about-the-new-covid-19-variants/
[8] Thomas, Philip, 2020, "The length and severity of the coronavirus recession estimated from the dynamics of relaxing lockdown", *Nanotechnology Perceptions*, http://www.colbas.org/ntp/opnAxs/N07TH20A.pdf

As shown in Figure 3, daily deaths fall rapidly, dropping to less than 100 a day by the middle of March 2021.

However, while the SDI rises to just below 1.9 by early autumn, its progress then stalls, with its value still less than half the SDI of 3.9 what is necessary for the full lifting restrictions – the country will remain in a highly restricted state of social distancing. See Figure 6. Nor is there any possibility of escaping the restrictions at any time in the future while the R-rate is kept at 0.6, despite the fact that all the adult population has now been vaccinated.

Figure 7 shows that vaccination immunity ends by rising to 30%, which is above the immunity generated by infection, 26%. But even when the T-cell immunity of 13% is added, the total population immunity is only 69% and, although sizeable, this is insufficient to allow all restrictions to be ended.

Coronavirus restrictions would need to remain in place permanently in England if the R-rate were to be kept constant at 0.6, even after all the adult population has been offered a vaccination.

4.2. The R-rate is held at 0.95 from the end of January 2021 onwards

Increasing the R-rate to 0.95 still allows both cases by date reported and active infections in England to decrease continuously, although more slowly than was the case when R = 0.6. See Figures 8 and 9.

Deaths per day fall away continuously, dropping to 200 a day by mid April 2021. The decrease becomes slower over the summer, but then starts to accelerate late August 2021 and deaths per day are numbered in the tens by the end of December 2021 (Figure 10).

Full derestriction is now possible, with the SDI reaching the necessary value of 3.9 that implies full opening up of society and the economy by mid August 2021. See Figure 11.

Crucially, the infection-generated immunity has risen to 40%, as shown in Figure 12. The vaccine immunity is now 25%, and the total immunity, in excess of 80%, is high enough to permit all restrictions to be lifted.

4.3. The R-rate is held at 1.1 from the end of January 2021 onwards

Increasing the R-rate to 1.1 will cause both cases by date reported and active infections in England to increase until the SDI reaches $\Sigma_0 = 3.9$ at the start of May 2021, at which point all restrictions are lifted (see Figure 13).

Figures 14 shows new daily cases rising to 70,000 at the beginning of May 2021, while Figure 15 shows the number of active infections surging to two million, almost double the previous peak of 1.1 million seen at the beginning of January 2021, although active infections drop rapidly thereafter, falling to below 100 by the end of November 2021.

But the vaccination exercise will have contained the daily deaths to a peak of about 600 per day in late May 2021, after which they fall away rapidly, dropping to below 10 by the start of September 2021 and zero by mid October.

Infection-generated immunity has rises to 57% eventually. But it should be noted that the number of new infections that lead to death has been very significantly reduced. The vaccine immunity plateaus at 21%, and the total immunity is 90%, which is significantly higher than is needed for all restrictions to be lifted.

5. Two-step approach

5.1 *R* = 0.6 initially and then all restrictions are lifted at the end of May 2021.

Figure 18 shows the step lifting of the SDI from 1.35 to 3.9 at the end of May 2021. This results in the cases by day reported starting gradually to increase, rising to over 20,000 a day in September 2021 before falling away at the end of November (Figure 19). Figure 20 shows the total number of active infections in England rising above 600,000 in September and then dying away over the next 6 months. But Figure 21 shows the associated increase in daily deaths is modest, with the number peaking at about 300 at the beginning of October 2021 before declining.

The principal component of the final population immunity of 80% is immunity generated by infection – 37%, with immunity deriving from vaccination making up 30% (Figure 22).

5.2 *R* = 0.95 initially and then all restrictions are lifted at the end of May 2021.

In this case the SDI has already reached 2.65 by the time the final step up occurs (Figure 23).

The cases by date reported rise to about 35,000 a day in early July and then retreat (Figure 24), and the number of active infections rises back up to about a million cases, comparable with early January 2021 (Figure 25). But the number of deaths per day is much lower now, peaking at about 350 in late July, as shown in Figure 26 – similar in magnitude to the number of additional daily cases when *R* = 0.6.

Population immunity levels out at 86%, the largest part of which (47%) is infection generated immunity. Vaccination produces 25% immunity. See Figure 27

6. Two-step strategy: sensitivity studies, where the SDI associated with full unlocking is $\Sigma_0 = 6$ and the 2nd step occurs at the end of July 2021.

6.1 *R* = 0.95 initially and then all restrictions are lifted at the end of July 2021.

The SDI now rises from 3.6 to 6 in a step at the end of July 2021 (Figure 28). This causes the cases by date reported to rise to 38,000 in mid September, a little higher than in the scenario of Section 5.2, as shown in Figure 29. The active infections in England also rise a bit to 1.07 million, see Figure 30. The deaths per day now peak at 500 at the end of September before fading away in the autumn of 2021, as illustrated in Figure 31.

Population immunity rises to 90%, with infection-generated, 51%, followed by vaccine-induced, 27%, the principal components.

It is striking how delaying the second step by two months has rendered the sensitivity scenario of this section so similar to the scenario of Section 5.2, despite the much greater infectivity assumed and the higher infection fatality rate.

The same does not apply when the initial value of the R-rate is kept constant at 0.6.

6.2 *R* = 0.6 initially and then all restrictions are lifted at the end of July 2021.

Figure 33 shows that the SDI now rises in a step from 1.6 to 6 at the end of July 2021. There is a delayed but very large rise in the number of cases by date reported, which reaches a peak of 86,000 at the beginning of November 2021, higher than England has seen (Figure 34). Meanwhile the active cases in England rise to 2.3 million in the middle of November 2021, double the highest seen previously, as shown in Figure 35. Deaths peak at 1200 a day at the end of November before falling back to a low level in February 2022, see Figure 36.

93% immunity is finally achieved, with 49% coming from recovery from infection and 30% vaccine generated.

7. Discussion

7.1 Gradualist approach

A key point emerging is that vaccination-generated immunity, while it can make a very valuable contribution to ending the crisis, should be regarded as only one component of the solution. There are two other components: prior, T-cell immunity and immunity generated by infection.

Scenarios that allow a full escape from restrictions require infection-generated immunity to be greater than vaccine-generated immunity. The latter can contribute between 20 and 30% of immunity, but infection generated immunity needs to supply at least 35%.

While the government and its advisers have generally emphasised the desirability of suppressing the virus or at least keeping it as low as possible, the analysis presented here suggests that this policy would prevent England emerging fully from lockdown when a gradualist approach is used. Keeping the R-rate well below 1.0 will lead to restrictions being kept in place indefinitely – an R-rate of about 0.9 represents the cut-off point under the gradualist approach; an R-rate maintained below this value would prevent escape.

Although there has been widespread criticism of the UK's alleged tardiness in locking down, the analysis here suggests that it is necessary for the infection to spread widely at some point in order for the crisis to be terminated. On the other hand, infections acquired after vaccination are to be preferred to those contracted in the absence of vaccination because they incur many fewer deaths. For while vaccination does not provide complete immunity from infection and hence does not fully prevent transmission (it is, of course, very helpful in this regard), it does give strong protection against dying from Covid-19.

Exiting the lockdown to the point where all restrictions can be eased fully while keeping the daily death rate below the high level seen in late January 2021 requires a narrow path to be followed for the six months between February and August 2021, keeping the R-rate in a central band around 1.0. Allowing the level of the virus in circulation to fall too low or to rise too high are both options to be avoided in the gradualist approach.

Close control of the R-rate is clearly needed and this will require the R-rate to be measured accurately, continuously and rapidly.

Keeping the R-rate close to 1.0 by gradually but continually relaxing restrictions would allow all restrictions to be lifted around the end of August 2021 under the gradualist approach.

7.2 The two-step strategy

The two-step strategy takes advantage of

(i) the fact that infections after vaccination are much less dangerous that those incurred pre-inoculation, including for the old and the vulnerable

(ii) vaccination is continuously decreasing the number of susceptible people and therefore the potential size of an uncontrolled epidemic.

Controlling the R-rate below 1.0 until the end of May 2021 and then opening up fully provokes a further surge of cases, both when the R-rate is maintained at 0.6 and when it is kept at 0.95. But this large but temporary increase is not accompanied by a huge increase in the number of deaths seen each day – a peak of roughly 350 deaths per day is experienced, but this declines fairly rapidly down to low levels.

The sensitivity study examined the case where the new UK variant become fully dominant over the original strain, leading to a higher infection fatality rate and a higher value of Σ_0. As a further test for the two-step strategies, the UK variant was assumed to possess twice the infectivity of the original strain, so that $\Sigma_0 = 6$. To compensate for this higher value, the final exit from lockdown was delayed by two months, from the end of May to the end of July 2021.

The two-step strategy with an initial R-rate kept constant at 0.95 was largely unaffected, in the sense that the peak daily death rate rose only a little higher. However a much larger effect was seen when the R-rate was maintained at 0.6 initially – the relative lack of prior infections meant that many more were now experienced and the daily deaths rose to the peak level seen in mid January 2021, namely 1200 per day. This pointed to the lack of robustness in the strategy if the country were kept locked down too hard.

8. The accuracy of the results

The PCCF model used in the analysis has demonstrated a good record in measuring the number of active infections in England by reconciling two disparate official measurements: the daily "cases by date recorded" produced by Public Health England and the ONS's coronavirus infection survey.[9] However the current analysis is projecting forward for a period of a year or more, which removes the feedback inherent in the two data sources: the PCCF is being used in forward prediction rather than in measurement mode.

Models help us comprehend the real world and they need to be simplifications of that world if they are to provide us with an understanding of which features are important to the problem in hand, a point expressed well by Finkelstein: "To treat effectively the complexity of real systems, the models used are highly abstract, that is to say they idealise and omit detail."[10] In this spirit, the figures quoted need be taken as indicative rather than exact. Nevertheless it is hoped and expected that the analysis will have captured the dominant features and modes of the problem of leaving lockdown.

9. Conclusions

While vaccination-generated immunity will undoubtedly make a strong contribution to ending the crisis, the other components of the solution are T-cell immunity and immunity generated by infection. Infection generated immunity needs to be the largest component if all restrictions are to be removed.

Keeping the R-rate below about 0.9 leads to restrictions being kept in place indefinitely if the gradualist approach is adopted, even after all the adult population has been vaccinated. Moreover, maintaining the R-rate too low in the initial phase of the two-step strategy low reduces its robustness against the possible future dominance of the new UK variant, B.1.1.7.

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[9] See *The Spectator* Covid-19 data tracker. https://data.spectator.co.uk/city/national
[10] Finkelstein, L., 2006, "From technology to wider knowledge, understanding and wisdom", Measurement and Control, Vol. 39, No. 9, 268 – 272.

Exiting the lockdown to the point where all restrictions can be eased fully requires a narrow path to be followed during the spring and early summer of 2021, keeping the R-rate in a central band around 1.0. Allowing the level of the virus in circulation to fall too low or to rise too high are both options to be avoided if the spirit of the UK government's policy on Covid-19 is to be retained.

Close control of the R-rate is needed and this will require the R-rate to be measured accurately, continuously and rapidly.

Keeping the R-rate close to 1.0 by gradually but continually relaxing restrictions might allow all restrictions to be lifted by the end of the summer, but the process of exiting might be speeded up by modifying the approach and using a two step strategy. This might allow full derestriction by the end of May 2021.

Appendix A. Modelling the disease response when the cohorts are divided into groups for vaccination according to age

Let the population of the country be split into two cohorts, i, i = 1, 2, each of which will be divided into 8 age groups, j: j = 1, ...,8. Anyone in Cohort 1 who contracts Covid-19 will display symptoms and will take a test. People in Cohort 2 contracting Covid-19 will experience either mild or no symptoms; such people will never be tested and hence their infection will never be reported. It is estimated that θ_1 = 30% of the population belongs to Cohort 1 while $\theta_2 = 1 - \theta_1$ = 70% of the population belongs to Cohort 2[11].

The age groups are listed in Table A.1 below.

The number of people, $N_i^{(j)}$, in age group, j, of cohort, i, will be:

$$N_i^{(j)} = p^{(j)} \theta_i N \qquad \text{(A.1)}$$

where $p^{(j)}$ is the fraction of the population in age group j.

The basic reproduction number is the number of potentially infective contacts made by the average person between infection and recovery in a fuller susceptible population. Infections will be transmitted if the contact is with a susceptible person. Each of the basic reproduction number, R_{0i}, and the average time between successive generations, $\tau_{\inf,i}$, will taken to be representative of cohort, i, across all age groups, j = 1, 2, ...,8.

At the start of vaccination, $t = t_{v0}$, and the fraction, $n_{si}^{(j)}(t_{v0}) / n_{si}(t_{v0})$, of susceptible people in age group j of cohort i to those of all ages across the cohort will be equal to the fraction $N_i^{(j)}(t_{v0}) / N_i(t_{v0}) = p^{(j)}$ of people of that age in the cohort, assuming a uniform distribution of infections across ages. Hence

$$n_{si}^{(j)}(t_{v0}) = p^{(j)} n_{si}(t_{v0}) \qquad \begin{matrix} i = 1, 2; \\ j = 1, 2, ..., 8 \end{matrix} \qquad \text{(A.2)}$$

This equation forms one starting condition for the vaccination exercise.

[Clearly the summing equation (A.2) over all j will give the number of susceptible people in the cohort:

$$\sum_{j=1}^{8} n_{si}^{(j)}(t_{v0}) = n_{si}(t_{v0}) \sum_{j=1}^{8} p^{(j)} = n_{si}(t_{v0}) \qquad i = 1, 2 \qquad \text{(A.3)}$$

[11] Thomas, P., 2020, Measuring and controlling the Covid-19 pandemic, Nanotechnology Perceptions, IN PRESS.

since $\sum_{j=1}^{8} p^{(j)} = 1$. Moreover, the total number of susceptible people in the population is found, of course, by adding together the contributions from the two cohorts:

$$n_s\left(t_{v0}\right) = n_{s1}\left(t_{v0}\right) + n_{s2}\left(t_{v0}\right) \qquad \text{(A.4)]}$$

Let $n_i^{(j)}\left(t\right)$ be the number of people with an active infection in the j^{th} age group in cohort i at time, t. At time, t_{v0}, just before vaccination starts, a fraction, $n_i^{(j)}\left(t_{v0}\right)/n_i\left(t_{v0}\right)$, of infectious people in age group j of cohort i will be equal to the fraction $N_i^{(j)}\left(t_{v0}\right)/N_i\left(t_{v0}\right) = p^{(j)}$ of people of that age in the cohort so that:

$$n_i^{(j)}\left(t_{v0}\right) = p^{(j)} n_i\left(t_{v0}\right) \qquad i = 1,2;\ j = 1,2,\ldots,8 \qquad \text{(A.5)}$$

This equation constitutes a second starting condition.

[By analogy with equations (A.4) and (A.5), summation gives the number of people with an active infection in each cohort:

$$\sum_{j=1}^{8} n_i^{(j)}\left(t_{v0}\right) = n_i\left(t_{v0}\right) \sum_{j=1}^{8} p^{(j)} = n_i\left(t_{v0}\right) \qquad i = 1,2 \qquad \text{(A.6)}$$

and in the whole population:

$$n\left(t_{v0}\right) = n_1\left(t_{v0}\right) + n_2\left(t_{v0}\right) \qquad \text{(A.7)]}$$

The rate at which meetings with the potential to pass on infection are taking place between an infected person in age group j of cohort i and some other individual is:

$$\frac{R_{0i} n_i^{(j)}}{\tau_{\text{inf},i}} \qquad \begin{matrix} i = 1,2; \\ j = 1,2,\ldots,8 \end{matrix}$$

The rate at which potentially infectious meetings are taking place between an infected person and someone, anyone, else is the sum over both cohorts and all age groups within them may be called the common "driver", D:

$$D = \sum_{i=1}^{2} \sum_{j=1}^{8} \frac{R_{0i} n_i^{(j)}}{\tau_{\text{inf},i}} \qquad \text{(A.8)}$$

If everyone in the population were susceptible, a so-called "naive" population, then this would represent the rate at which people were becoming infected. Their infection could come from either of the cohorts and from someone of any age within either cohort.

However, as a result of natural immunity, previous infection and vaccination, the number of susceptible people will be less than 100% of the population. Given that one of these potentially infective meetings has happened, the probability of someone in age group *j* of cohort *i* then being infected is simply the probability that the meeting happens between an infected individual and someone susceptible in age group, *j*.

The meeting might involve anyone in the population, and so the chance of the encounter being with someone susceptible in age group, *j*, is the number, $n_{si}^{(j)}$ of such people divided by the number of people in the population at large less the infected person, viz. $N-1$. Of course $N-1 \to N$ as *N* gets very big, as is the case here, where each age group contains millions of people. Hence we may write the probability as $n_{si}^{(j)}(t)/N$.

Utilising the driver concept introduced in equation (A.8), the rate, $dn_{xi}^{(j)}/dt$, at which people in age group *j* in cohort, *i*, are being infected will be

$$\frac{dn_{xi}^{(j)}}{dt}(t) = \frac{n_i^{(j)}}{N} D \qquad \text{(A.9)}$$

which may be re-expressed as

$$\frac{dn_{xi}^{(j)}}{dt}(t) = \frac{n_{si}^{(j)}(t)}{N}\left(R_{01}\sum_{j=1}^{8}\frac{n_1^{(j)}(t)}{\tau_{\inf,1}} + R_{02}\sum_{j=1}^{8}\frac{n_2^{(j)}(t)}{\tau_{\inf,2}} \right) \qquad \text{(A.10)}$$

Equations (A.8) and (A.9) make the cross-infection between groups explicit.

The rate at which infected people in age group *j* of cohort *i* pass on their infection per day and then recover or die may be called the rate of pure recovery, $\left(dn_{ri}^{(j)}/dt\right)\Big|_{pure}$, and given by (cf. equation (A.9) in Thomas (2020)[12])

$$\left.\frac{dn_{ri}^{(j)}}{dt}\right|_{pure} = \frac{n_i^{(j)}(t)}{\tau_{\inf,i}} \qquad \begin{array}{l} i = 1,2; \\ j = 1,2,\ldots,8 \end{array} \qquad \text{(A.11)}$$

This may be subtracted from the rate at which people are infected to give the net rate of growth of active infections in age group *j* of cohort *i*:

[12] Thomas, P., 2020, "J-value assessment of how best to combat COVID-19", *Nanotechnology Perceptions*, Vol.16, pp. 16–40, available at: http://www.colbas.org/ntp/opnAxs/N02TH20A.pdf

$$\frac{dn_i^{(j)}}{dt}(t) = \frac{dn_{xi}^{(j)}}{dt}(t) - \left.\frac{dn_{ri}}{dt}\right|_{pure} \qquad i = 1,2;$$
$$== \frac{n_{si}^{(j)}(t)}{N}\left(R_{01}\sum_{j=1}^{8}\frac{n_1^{(j)}(t)}{\tau_{\inf,1}} + R_{02}\sum_{j=1}^{8}\frac{n_2^{(j)}(t)}{\tau_{\inf,2}}\right) - \frac{n_i^{(j)}(t)}{\tau_{\inf,i}} \qquad j = 1,2,...,8 \qquad \text{(A.12)}$$

It is convenient, in the model, to collect under the heading, "recovered":

(i) those who had the illness but are now well again – the "pure" recovered,
(ii) those with prior, T-cell immunity, estimated to be 12.9% of the population,
(iii) those who have been vaccinated and whose inoculation was long enough ago (τ_v) to be effective,
(iv) those who, unfortunately, will succumb to the disease.

It is assumed that, just before vaccination starts, the fraction of people recovered who fall into age group *j* of any cohort *i* will be approximately proportional to the fraction, $p^{(j)}$, of people of that age group in the population:

$$n_{ri}^{(j)}(t_{v0}) = p^{(j)} n_{ri}(t_{v0}) \qquad i = 1,2; \quad j = 1,2,...,8 \qquad \text{(A.13)}$$

The rate of change in the number of people recovering will not be affected by the phenomenon of prior, T-cell immunity, since this number will not change during the course of the epidemic.

However, rate of change in the number of people recovering will depend both the rate of pure recovery, given by equation (A.10). and the rate of vaccination maturity.

Allowing for vaccination

The event of vaccination has strong similarities to that of infection. In this case, however, what results is future immunity, rather than the onset of disease – "future" immunity because protection is given only if the person avoids exposure to infection within the roughly three weeks after inoculation that allow the vaccine shot to become fully effective. This future immunity will not be conferred to a person who is already infected or has already recovered nor to those with prior T-cell immunity. New immunity can only be given if the vaccine is administered to a susceptible person.

We define an eligible, susceptible person as a person who is susceptible to the disease but who has not yet been vaccinated. At the start of vaccination, when $t = t_{v0}$, the number of eligible susceptible people is the same as the number of susceptible people:

$$n_{esi}^{(j)}(t_{v0}) = n_{si}^{(j)}(t_{v0}) \qquad \text{(A.14)}$$

But while the rate of change, $dn_{si}^{(j)} \big/ dt$, of susceptible people in age group, *j*, in Cohort *i* is simply the rate at which people of that age group and cohort are being infected, viz. $dn_{si}^{(j)} \big/ dt = -dn_{xi}^{(j)} \big/ dt$, the rate of change of eligible susceptible people will tend to be greater, as the people can leave the ranks of those susceptible and eligible by either
(i) becoming infected, when a fraction, $n_{esi}^{(j)} \big/ n_{si}^{(j)}$, of $dn_{xi}^{(j)} \big/ dt$ will come from the eligible subset of the susceptible; or else
(ii) by being vaccinated at rate, $v_{esi}^{(j)}$.

Hence

$$\frac{dn_{esi}^{(j)}}{dt} = -\frac{n_{esi}^{(j)}}{n_{si}^{(j)}} \frac{dn_{xi}^{(j)}}{dt} - v_{esi}^{(j)} \qquad \text{(A.15)}$$

Here $v_{esi}^{(j)}$ is the rate of vaccination of eligible, susceptible people in age group *j* and cohort *i*. This may be calculated as

$$v_{esi}^{(j)}(t) = p_{vesi}^{(j)}(t) v^{(j)}(t) \qquad \text{(A.16)}$$

where $v^{(j)}(t)$ is the rate at which people are being vaccinated in age group *j*. Meanwhile $p_{vesi}^{(j)}$ is the likelihood of selecting an eligible, susceptible person who belongs to age group *j* and cohort *i* , to be included in the day's age-dependent vaccination batch; that is to say choosing someone who (i) belongs to that grouping, (ii) is susceptible and (iii) has not yet been vaccinated.

The government specifies and arranges the rates of vaccination, $v^{(j)}(t)$, in age groups first to those aged 80 and over first, then 70 to 79 year olds and so on. It is assumed that everyone in the age group *j*, across both cohorts, who has not yet been vaccinated stands the same chance of being selected for inoculation. Those who are immune through recovery or through prior T-cell protection will be eligible for vaccination *pari passu* with those who are actually susceptible.

Let the number of people yet to be vaccinated in age group *j* be $m^{(j)}(t)$, which defines the size of the full vaccination pool across both cohorts for age *j*.

(Strictly, it is unlikely that those showing a current infection will be injected, which implies a fraction, *c*, of those in Cohort 1 with an active infection, so $cn_1^{(j)}(t)$, where $0 < c < 1$ is the strictly proper fraction of the time between infection and recovery for which symptoms are clearly visible. This would reduce the vaccination pool to $m^{(j)}(t) - cn_1^{(j)}(t)$, but since $m^{(j)}(t) >> cn_1^{(j)}(t)$ for most of the time, little error is introduced by omitting the second term.)

Let us denote by $n_{esi}^{(j)}(t)$ the current number of eligible susceptible people in age group *j* of cohort *i*. The likelihood, $p_{vesi}^{(j)}$, of selecting for the vaccination campaign an eligible, susceptible person who belongs to age group *j* and cohort *i* will be the ratio of this number, $n_{esi}^{(j)}(t)$, to the total number of eligible people in the age group:

$$p_{vesi}^{(j)} = \frac{n_{esi}^{(j)}(t)}{m^{(j)}(t)} \qquad \text{(A.17)}$$

The starting condition for $m^{(j)}$ before vaccination starts is the number of people in age group *j* in the population: $m^{(j)}(t_{v0}) = N^{(j)}(t_{v0}) = p^{(j)}N$. It will, however, decrease as more people of age group *j* are vaccinated:

$$m^{(j)}(t) = m^{(j)}(t_{v0}) - y^{(j)}(t) \qquad \text{(A.18)}$$

where $y^{(j)}(t)$ is the number of people of age group *j* who have been vaccinated, found by integrating equation (A.19) below:

$$\frac{dy^{(j)}}{dt}(t) = -v^{(j)}(t) \qquad \text{(A.19)}$$

The vaccine efficacy at preventing disease and transmission, η_v, will grow from after the first injection and continue further after the second injection 12 weeks later. The process of vaccine maturation may be modelled as an exponential lag, obeying the equation:

$$\eta_v = \eta_{vf}\left(1 - e^{-\frac{x}{\tau_{mat}}}\right) \qquad \text{(A.20)}$$

where *x* is the time since first vaccination, η_{vf} is the final vaccine efficiency achieved after the second injection and τ_{mat} is the vaccine maturation time constant. The effect may be simulated by applying the final vaccine efficacy to the flow of vaccinated eligible and susceptible people, $v_{esi}^{(j)}$, and subjecting this to a first-order exponential lag:

$$\frac{dq_i^{(j)}}{dt} = \frac{n_{vf} v_{esi}^{(j)} - q_i^{(j)}}{\tau_{mat}} \qquad \text{(A.21)}$$

Integrating equation (A.22) gives the flow, $q_i^{(j)}(t)$, of people in age group *j* and cohort *i* becoming immune at time *t* as a result of vaccination.

This allows the rate of increase in recovered people in age group *j* and cohort *i* to be calculated:

$$\begin{aligned} \frac{dn_{ri}^{(j)}}{dt} &= \left.\frac{dn_{ri}^{(j)}}{dt}\right|_{pure} + q_i^{(j)}(t) \\ &= \frac{n_i^{(j)}(t)}{\tau_{\inf,i}} + q_i^{(j)}(t) \end{aligned} \qquad \begin{aligned} i &= 1,2; \\ j &= 1,2,\ldots,8 \end{aligned} \qquad \text{(A.22)}$$

In this model, a person of age *j* in cohort *i* will belong to one of the categories: susceptible, infected (with an active infection) and recovered. The time-marching integration of equations (A.11) and (A.22) will give the number of people in each of the classifications, "infected" and "recovered", while the total number in the age group and cohort, $N_i^{(j)}$, is given by equation (A.1) as $\theta_i p^{(j)} N$. Hence the number of susceptible people make be found by subtraction:

$$n_{si}^{(j)}(t) = \theta_i p^{(j)} N - n_i^{(j)}(t) - n_{ri}^{(j)}(t) \qquad \begin{aligned} i &= 1,2; \\ j &= 1,2,\ldots,8 \end{aligned} \qquad \text{(A.23)}$$

Vaccine protection against death

In addition to protecting against illness, with efficacy, vaccines will protect against dying from Covid-19 with eventual efficacy, η_{ND}. Early evidence suggests that this figure might be as high as 100% for the AstraZeneca vaccine, but in this study, η_{ND} is set more cautiously at 95% .

So while some people are protected against both disease and death by vaccination, a further set are protected against death. The rate, at which the number, $g_i^{(j)}$, of people in age group *j* and cohort *i* become protected, at time *t*, from dying as a result of vaccination but not against infection, may be modelled (c.f. equation (A.22)) as

$$\frac{dg_i^{(j)}}{dt} = \frac{\left(\eta_{ND} - n_{vf}\right) v_{esi}^{(j)} - g_i^{(j)}}{\tau_{mat}} \qquad \text{(A.24)}$$

Integrating equation (A.24) gives the flow, $g_i^{(j)}(t)$, of people protected against dying but still susceptible to infection.

Some of these people will become infected in the same way as other susceptible people in their age group and cohort. Hence the fraction of the flow of people being infected belonging to this "non-dying" category will be $\left.n_{si}^{(j)}\right|_{ND} \Big/ n_{si}^{(j)}$, where $\left.n_{si}^{(j)}\right|_{ND}$ is the number of susceptible people who are protected by vaccination against dying but not against infection. This number, $\left.n_{si}^{(j)}\right|_{ND}$, will therefore obey the differential equation:

$$\frac{d}{dt}\left.n_{si}^{(j)}\right|_{ND} = g_i^{(j)} - \frac{\left.n_{si}^{(j)}\right|_{ND}}{n_{si}^{(j)}}\frac{dn_{xi}^{(j)}}{dt} \qquad \text{(A.25)}$$

The number, $\left.n_{si}^{(j)}\right|_{D}$, of susceptible people who are not protected from dying may be found as:

$$\left.n_{si}^{(j)}\right|_{D} = n_{si}^{(j)} - \left.n_{si}^{(j)}\right|_{ND} \qquad \text{(A.26)}$$

The death rate

Let the probability of unvaccinated people in age group, *j*, dying of the disease if they catch it be $p_{D0}^{(j)}$. At time, *t*, the number of susceptible people in age group *j* will be the total across both cohorts $n_{s1}^{(j)}(t) + n_{s2}^{(j)}(t)$. Of those, there will be only $\left.n_{s1}^{(j)}\right|_{D} + \left.n_{s2}^{(j)}\right|_{D}$ people who may die if they catch Covd-19, with a probability of death of $p_{D0}^{(j)}$. Hence the overall probability of death once Covid-19 is contracted will be

$$\begin{aligned} p_D^{(j)}(t) &= \frac{0 \times \sum_{i=1}^{2} \left.n_{si}^{(j)}\right|_{ND} + p_{D0}^{(j)} \sum_{i=1}^{2} \left.n_{si}^{(j)}\right|_{D}}{\sum_{i=1}^{2} n_{si}^{(j)}} \\ &= \frac{\left.n_s^{(j)}\right|_{D}}{n_s^{(j)}} p_{D0}^{(j)} \end{aligned} \qquad \text{(A.27)}$$

The overall probability of dying across all age groups is then

$$p_D(t) = \frac{\sum_{j=1}^{8} n_s^{(j)}(t)\, p_D^{(j)}(t)}{\sum_{j=1}^{8} n_s^{(j)}(t)} \qquad \text{(A.28)}$$

Appendix B. Modelling the vaccination campaign prioritised according to age

Let the number of unvaccinated people in each age group across both cohorts be $m^{(j)}(t)$: $j = 1, 2, 3, ..., 8$. The values at the start of the vaccination process are then $m^{(j)}(t_{v0}) = p^{(j)}N$, where $p^{(j)}$ is the fraction of the population in age group *j*, j = 1,2,...,8 (see Table A.1).

The number of vaccinations, $v(t_n)$, on day, t_n, across all age groups is regarded as an exogenous variable. It follows the daily rates of first doses reported up to 6 February 2021, and is then set at the constant rate of 400,000 per day for the remainder of the vaccination campaign.

From 9 December 2020 to 18 January 2021, 60% of the vaccine doses are assumed distributed to those aged 80 and over and 6.7% each to those in age groups 70-79, 65-69, 60-64, 55-59, 50-54 and 18-49 to account for the vaccination of health workers and those vulnerable with morbidities.

From 19 January 2021 to 28 January 2021, 50% of the vaccine first doses are assumed to go to those of 80 and over and the remainder to those aged between 70 and 79. Vaccination of 92% of those aged 80 and over is completed on 29 January 2021.

Thereafter people are vaccinated in ordered age groups, starting with the completion of inoculation of group 2 and ending with group 7, with no vaccinations being given to children, age group 8, in line with government policy. Each successive group is vaccinated until only a fraction, f_{unvac} (set to 8% in this work), is left unvaccinated in each group.

Let the cumulative number of vaccinations given to age group, *j*, by day, t_n, be $y^{(j)}(t_n)$, where

$$y^{(j)}(t_n) = \int_{t_{v0}}^{t_n} v^{(j)}(t)\,dt \qquad \text{(B.1)}$$

where $v^{(j)}(t)$ is the number of vaccinations made on day, *t*, to members of age group, *j*.

The number of vaccinations administered to age group *j* on day t_n is then

$$v^{(j)}\left(t_n\right)=\begin{cases} v\left(t_n\right)-\sum_{k=2}^{j-1} v^{(k)}\left(t_n\right) & \text{if} \quad v\left(t_n\right)-\sum_{k=2}^{j-1} v^{(k)}\left(t_n\right)<y^{(j)}\left(t_{n-1}\right)-f_{unvac} m^{(j)}\left(t_{v0}\right) \\ y^{(j)}\left(t_{n-1}\right)-f_{unvac} m^{(j)}\left(t_{v0}\right) & \text{if} \quad v\left(t_n\right)-\sum_{k=2}^{j-1} v^{(k)}\left(t_n\right) \geq y^{(j)}\left(t_{n-1}\right)-f_{unvac} m^{(j)}\left(t_{v0}\right) \end{cases}$$

(B.2)

.

Appendix C. The death probabilities for different age bands in the absence of vaccination

Let the number of people in the population be N and the number in age group, j, be $N^{(j)}$, where the probability of being in age group, j, is

$$p^{(j)} = \frac{N^{(j)}}{N} \qquad \text{(C.1)}$$

Let the number of people in age group, j, recorded as dying from Covid-19 in the UK in 2020 be $n_D^{(j)}$, with a total number across all age groups:

$$n_D = \sum_j n_D^{(j)} \qquad \text{(C.2)}$$

Let us assume that everyone has the same chance, p_{inf}, of being infected. Hence the expected value of the (random) number, $N_{\text{inf}}^{(j)}$, of people infected in age group, j, is

$$E\left(N_{\text{inf}}^{(j)}\right) = p_{\text{inf}} N^{(j)} = p_{\text{inf}} p^{(j)} N \qquad \text{(C.3)}$$

Let the probability of people in age group, j, dying after being infected be $p_{D0}^{(j)}$. The expected number of people in age group, i, dying is therefore:

$$n_D^{(j)} = E\left(N_D^{(j)}\right) = p_{D0}^{(j)} E\left(N_{\text{inf}}^{(j)}\right) = p_{D0}^{(j)} p^{(j)} p_{\text{inf}} N \qquad \text{(C.4)}$$

Meanwhile the expected total number of deaths across all age groups may be calculated as

$$n_D = E\left(N_D\right) = p_{D0} E\left(N_{\text{inf}}\right) = p_{D0} p_{\text{inf}} N \qquad \text{(C.5)}$$

where p_{D0} is the overall probability of dying from Covid-19, given one has been infected for the population as a whole.

Dividing equation (C.4) by equation (C.5) gives:

$$\frac{p_{D0}^{(j)}}{p_{D0}} = \frac{1}{p^{(j)}} \frac{n_D^{(j)}}{n_D} \qquad \text{(C.6)}$$

or

$$p_{D0}^{(j)} = \frac{p_{D0}}{p^{(j)}} \frac{n_D^{(j)}}{n_D} \qquad \text{(C.7)}$$

Appendix D. Identifying the infection maturity time constant, τ_{mat}

Information provided by Oxford University scientists[13] showed that all infections were reduced by 67% for the period 22 to 90 days after a single dose of the Oxford-AstraZeneca vaccine. Assuming, conservatively, that no immunity builds up until after 21 days, the measured efficiency in preventing transmission may be modelled as:

$$\eta_{vm}(x) = \begin{cases} 0 & \text{for } 0 < x \le 21 \\ 0.67 & \text{for } 21 < x \le 90 \end{cases} \qquad \text{(D.1)}$$

where x is the number of days after the first injection.

The average efficiency from 0 to 90 days is therefore:

$$\begin{aligned} \eta_{vmeas}\big|_{ave} &= \frac{1}{90}\left(\int_{x=0}^{90} \eta_{vm}\,dx\right) = \frac{1}{90}\left(\int_{x=0}^{21} 0dx + \int_{x=22}^{90} 0.67dx\right) \\ &= \frac{0.67}{90} \times 69 \end{aligned} \qquad \text{(D.2)}$$

Modelling the process as a first order exponential lag produces equation (A.20), repeated below.

$$\eta_v = \eta_{vf}\left(1 - e^{-\frac{x}{\tau_{mat}}}\right) \qquad \text{(A.20)}$$

The average value of this over the period from 0 to 90 days will be

$$\begin{aligned} \eta_v\big|_{ave} &= \frac{\eta_{vf}}{90}\int_{x=0}^{90}\left(1 - e^{-\frac{x}{\tau_v}}\right)dx = \frac{\eta_{vf}}{90}\left(90 + \tau_v\left[e^{-\frac{x}{\tau_v}}\right]_0^{90}\right) \\ &= \frac{\eta_{vf}}{90}\left(90 + \tau_v e^{-\frac{90}{\tau_v}} - \tau_v\right) \end{aligned} \qquad \text{(D.3)}$$

Choosing the asymptotic value of η_v as $x \to \infty$ as $\eta_{vf} = 0.67$, the two averages of equations (D.22) and (D.23) will be equal if and only if:

$$90 + \tau_v e^{-\frac{90}{\tau_v}} - \tau_v = 69 \qquad \text{(D.4)}$$

[13] Voisey, M., et al., 2021, Single dose administration, and the influence of the timing of the booster dose on immunogenicity and efficacy of ChAdOx1 nCoV-19 (AZD1222) vaccine, paper submitted to The Lancet, 2 February, https://papers.ssrn.com/sol3/papers.cfm?abstract_id=3777268

which may be rearranged into the form:

$$\tau_v\left(1-e^{-\frac{90}{\tau_v}}\right)-21=0 \qquad \text{(D.5)}$$

Inspection of equation (D.5) suggests that the time constant will be close to 21 days (the exponential term becomes very small), and an iterative solution yields:

$$\tau_v = 21.31 \text{ days} \qquad \text{(D.6)}$$

Appendix E. Allowing for differences in test numbers: development of an alternative method

Appendix D of the 4th J-value coronavirus paper explains the basis for estimating the number of cases there would have been if there had been the same number of tests on a given day as there were on 13 November 2020. See Table E.1.

Test and trace data give weekly estimates in arrears of the fractions, p_1 and p_2, of positives in the two pillars, and these have been used to generate an estimate of the number of positive cases standardised to the tests of 13 November 2020 according to the following procedure.

The difference in Pillar 1 tests on the day, w_1, and the standardised number, w_{10}, will be

$$\Delta w_1 = w_1 - w_{10} \qquad \text{(E.1)}$$

The expected excess positives will be

$$E\left(\Delta M_1\right) = p_1\left(w_1 - w_{10}\right) \qquad \text{(E.2)}$$

Given an actual number of positive tests or cases in Pillar 1 on the day, M_1, the standardised number is calculated as

$$M_{1\text{stan}} = M_1 - E\left(\Delta M_1\right) \qquad \text{(E.3)}$$

Similar equations hold for Pillar 2:

$$\Delta w_2 = w_2 - w_{20} \qquad \text{(E.4)}$$

$$E\left(\Delta M_2\right) = p_2\left(w_2 - w_{20}\right) \qquad \text{(E.5)}$$

$$M_{2\text{stan}} = M_2 - E\left(\Delta M_2\right) \qquad \text{(E.6)}$$

The overall standardised figure is then the summation:

$$\begin{aligned} M_{\text{stan}} &= M_1 + M_2 - E\left(\Delta M_1\right) - E\left(\Delta M_2\right) \\ &= M - E\left(\Delta M_1\right) - E\left(\Delta M_2\right) \end{aligned} \qquad \text{(E.7)}$$

or

$$M_{\text{stan}} = M - p_1\left(w_1 - w_{10}\right) - p_2\left(w_2 - w_{20}\right) \qquad \text{(E.8)}$$

Alternative method

The approach is now adapted to produce an alternative method as follows.

Calculate the probabilities from the day's figures for Cases by Date Reported and Reported Tests under each Pillar, using the assumption that the new estimated probabilities under the two pillars, p_1^* and p_2^*, are related by

$$\frac{p_1^*}{p_2^*} = r_p = \frac{p_1}{p_2} \qquad \text{(E.9)}$$

where p_1 and p_2 are the values from Test and Trace data. There will be an inaccuracy caused by the fact that the figure applies to the previous week, but the change is not likely to be large.

The expected value of the number of positives or cases is then

$$E(M) = p_1^* w_1 + p_2^* w_2 = r_p p_2^* w_1 + p_2^* w_2 = \left(r_p w_1 + w_2\right) p_2^* \qquad \text{(E.10)}$$

Then using *M* as an estimator for $E\left(M\right)$, it follows that:

$$p_2^* \approx \frac{M}{r_p w_1 + w_2} \qquad \text{(E.11)}$$

while, applying equation (E.9) to equation (E.11) gives:

$$p_1^* \approx \frac{r_p M}{r_p w_1 + w_2} \qquad \text{(E.12)}$$

The standardised figure is then

$$M_{\text{stan}} \approx p_1^* w_{10} + p_2^* w_{20} \qquad \text{(E.13)}$$

$$M_{\text{stan}} \approx r_p M \frac{w_{10}}{r_p w_1 + w_2} + M \frac{w_{20}}{r_p w_1 + w_2} \qquad \text{(E.14)}$$

$$M_{\text{stan}} \approx M \frac{r_p w_{10} + w_{20}}{r_p w_1 + w_2} \qquad \text{(E.15)}$$

This method has the advantage of relying only on the ratio of the probabilities in Pillars 1 and 2, not on the absolute values of both. This is expected to change more slowly from week to week, thus allowing a smoother set of estimates to develop.

Tables

Age group identifier, *j*	**Age group (years)**	**Fraction of English population in age group, $p^{(j)}$**	**Number in age group across both cohorts $m^{(j)}(0)$**
1	80 – 100	7.56%	4,272,973
2	70 – 79	9.35%	5,282,551
3	65 – 69	5.33%	3,012,633
4	60 – 64	5.59%	3,156,411
5	55 – 59	5.75%	3,251,219
6	50 – 54	5.86%	3,313,718
7	18 – 49	38.60%	21,809,057
8	0 – 17	21.95%	12,401,437
	0 – 100	100%	56,500,000

Table A.1. Age groups and their identifiers

Reference Date	Tests in Pillar 1, w_{10}	Tests in Pillar 2, w_{20}	Total tests across both Pillars, w_0
13-Nov-20	71,749	250,067	321,816

Table E.1. Reference date with reference numbers of tests in Pillars 1 (hospitals) and 2 (general public)

Vaccine-mediated exit strategies from England's Covid-19 lockdown: Figures

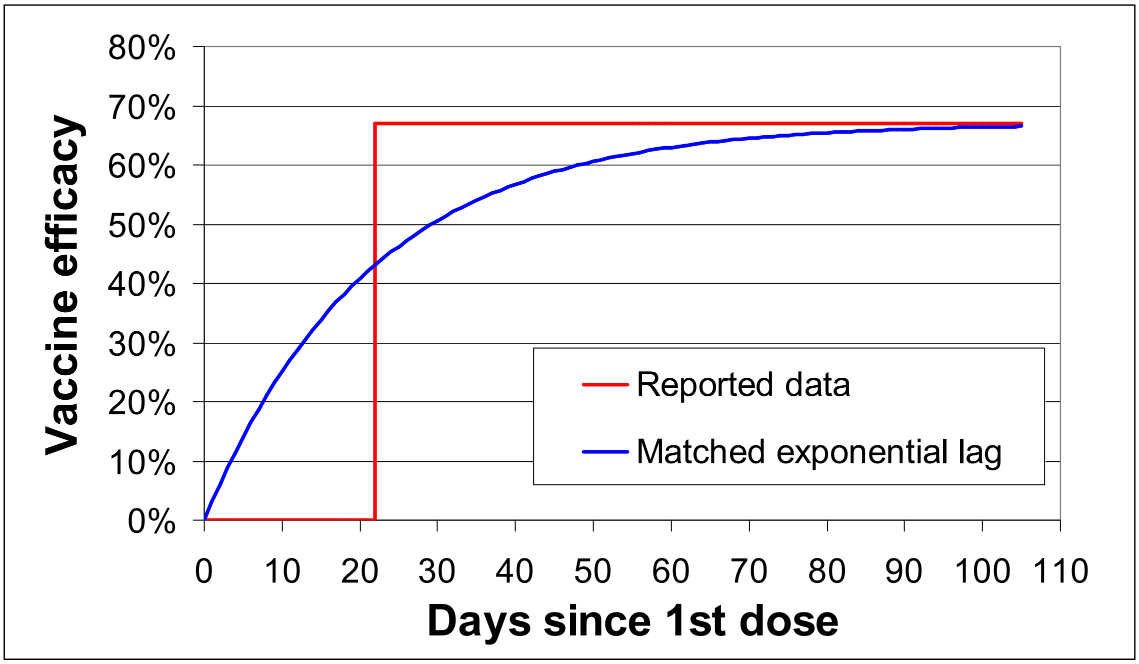

Figure 1. Matching an exponential lag to reported vaccine effectiveness

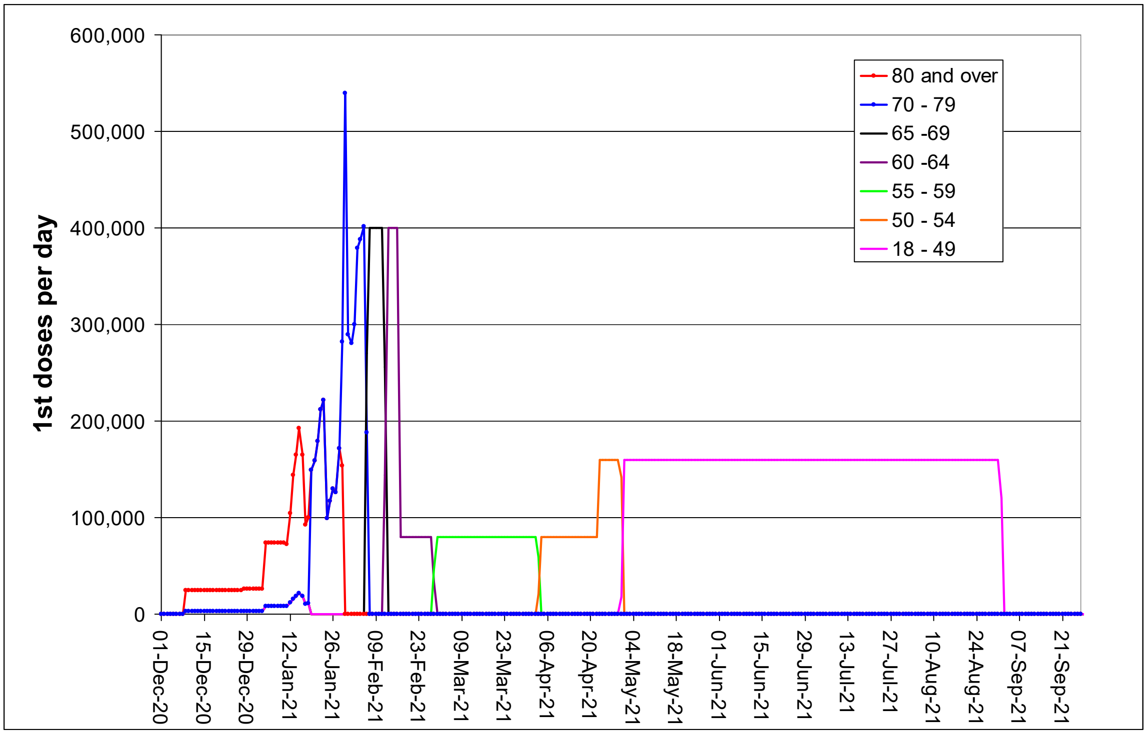

Figure 2. Programme of doses administered to the 7 age-groups for adults in England

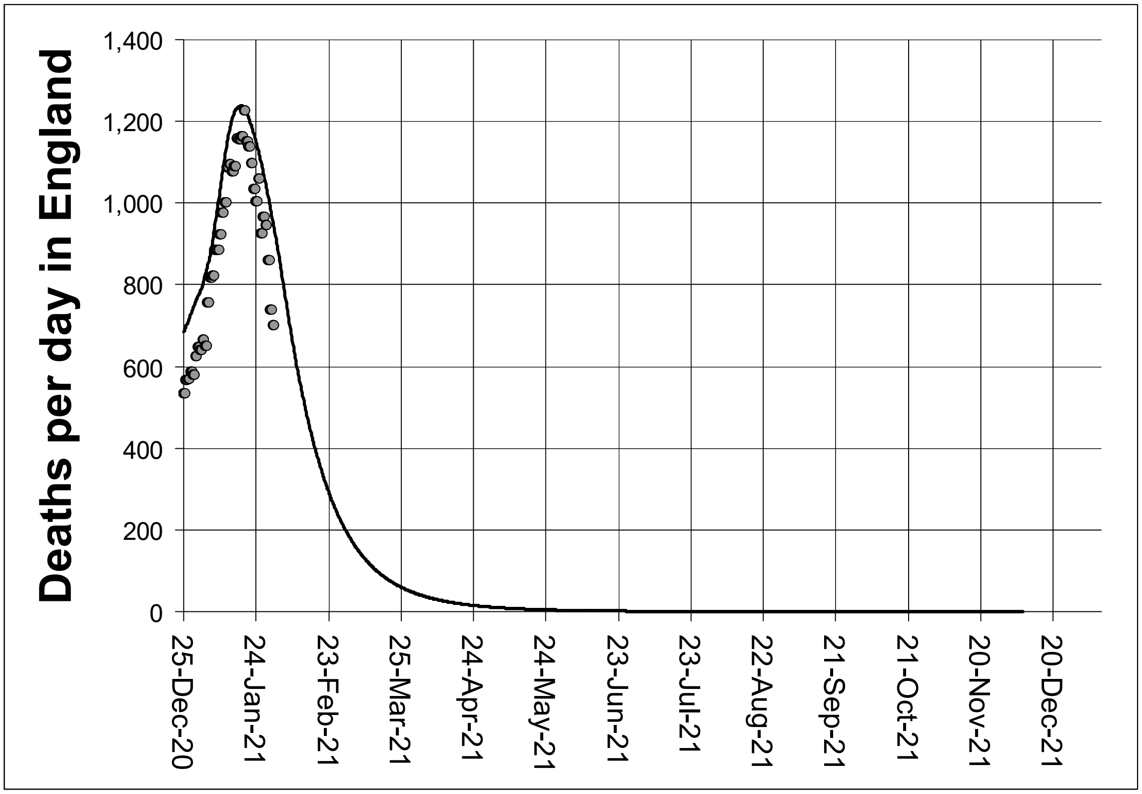

Figure 3. Adjusting the model's infection fatality rate to match the prediction to the peak daily deaths recorded in England in January.
(The R-rate is held at 0.6 in the model from the end of January onwards, leading to the continuing decline in deaths per day.)

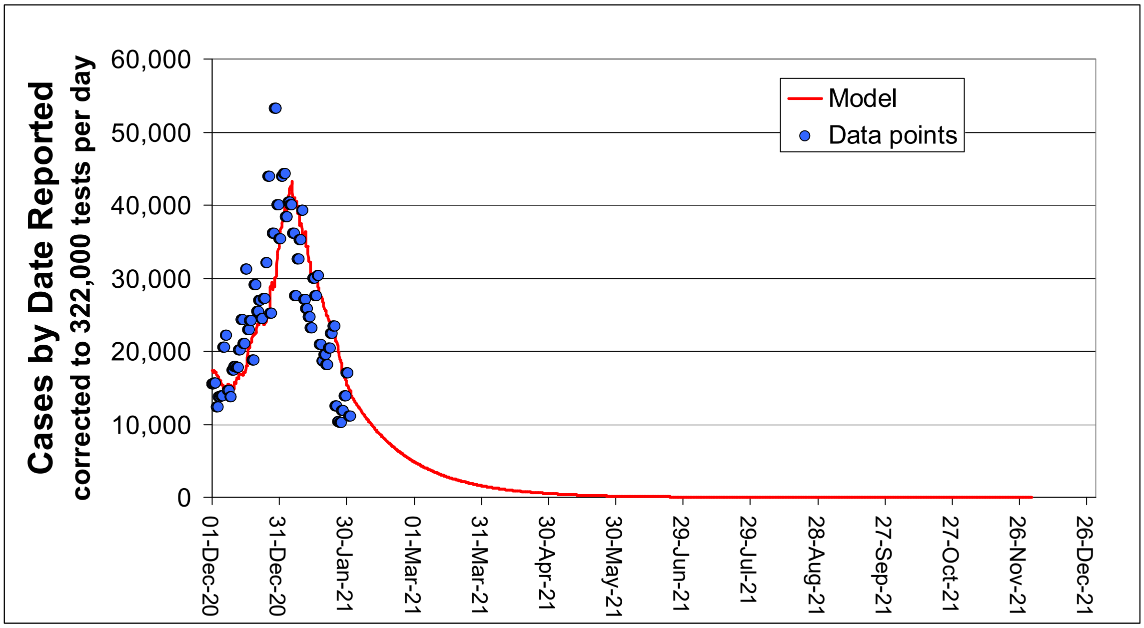

Figure 4. The effect on cases by date reported of keeping the R-rate at 0.6 from the end of January 2021 onwards

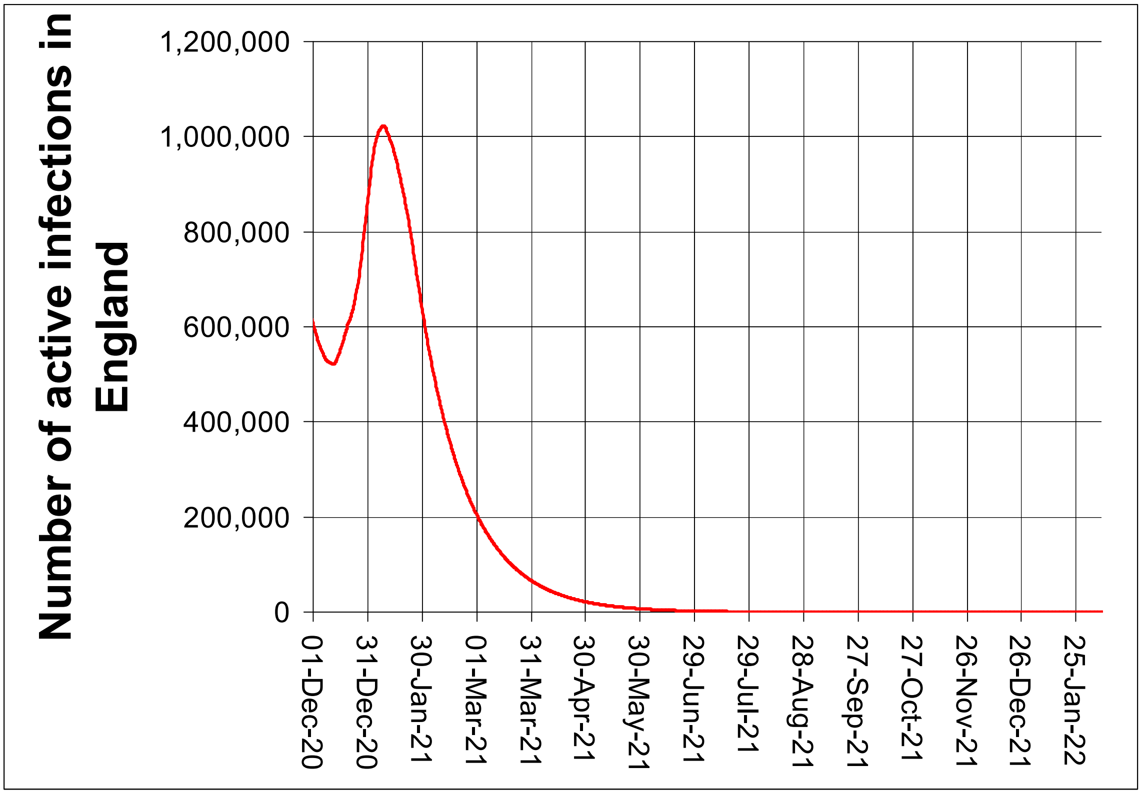

Figure 5. Active infections in England. *R* = 0.6

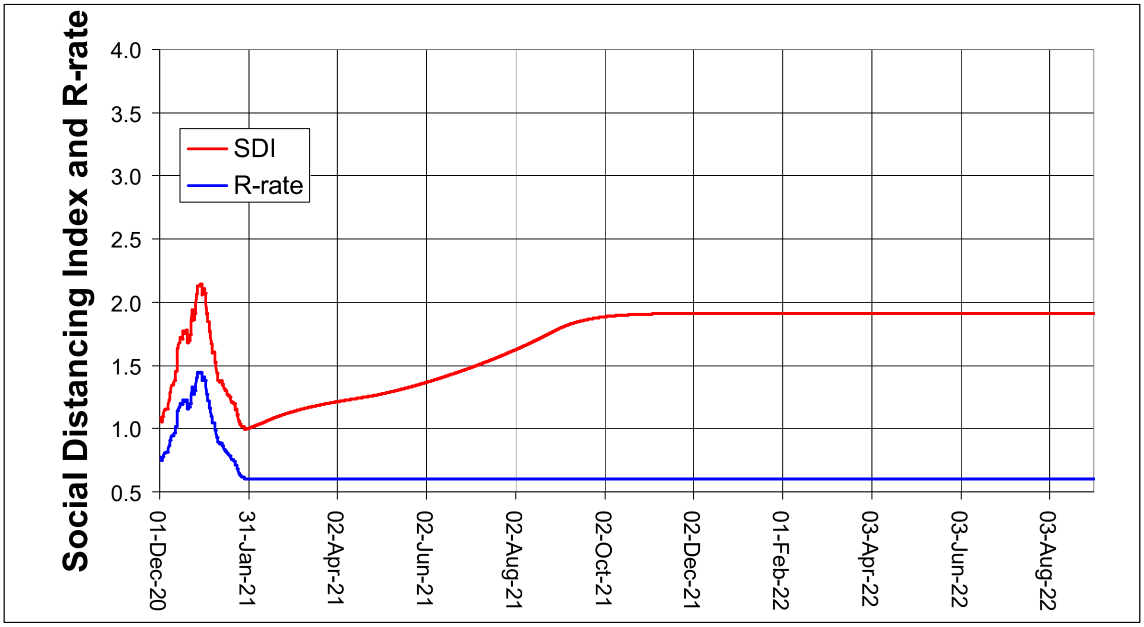

Figure 6. Social Distancing Index (SDI) and R-rate; *R* = 0.6

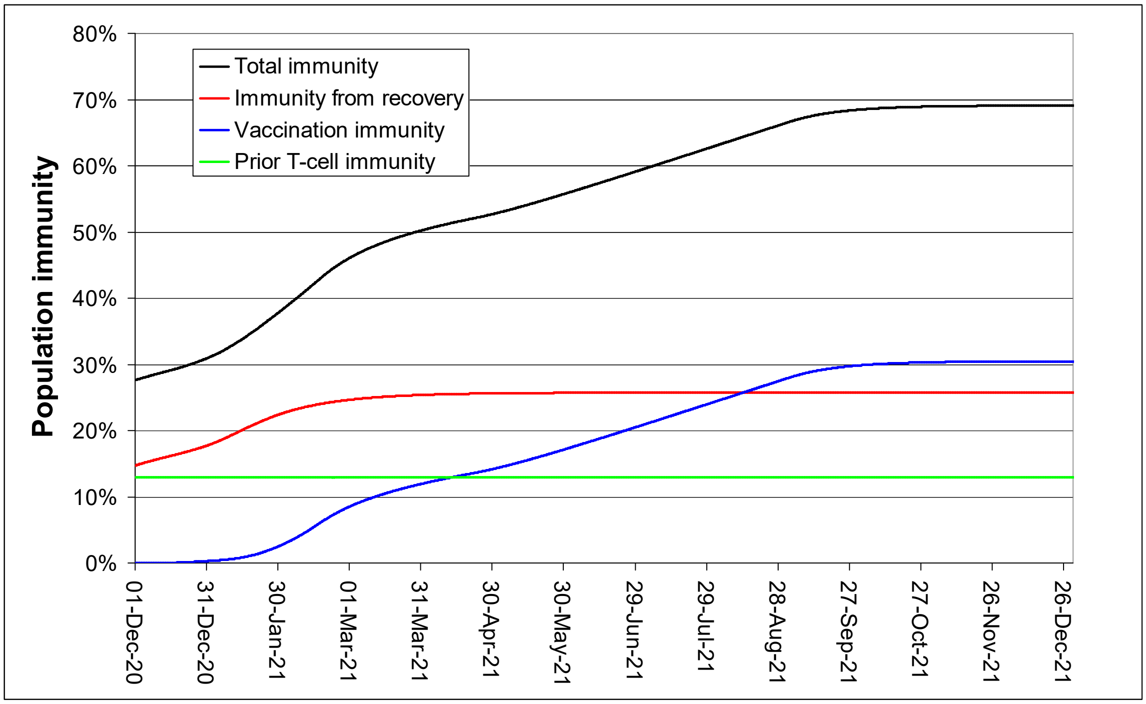

Figure 7. Components of population immunity, *R* = 0.6

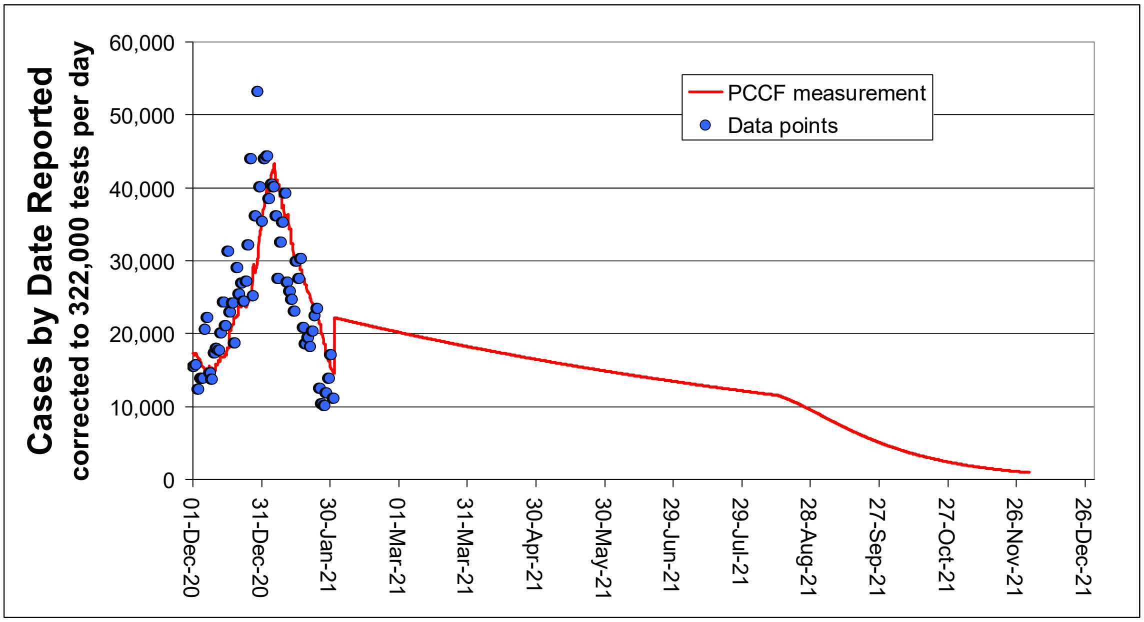

Figure 8. The effect on cases by date reported of keeping the R-rate at 0.95 from the end of January 2021 onwards

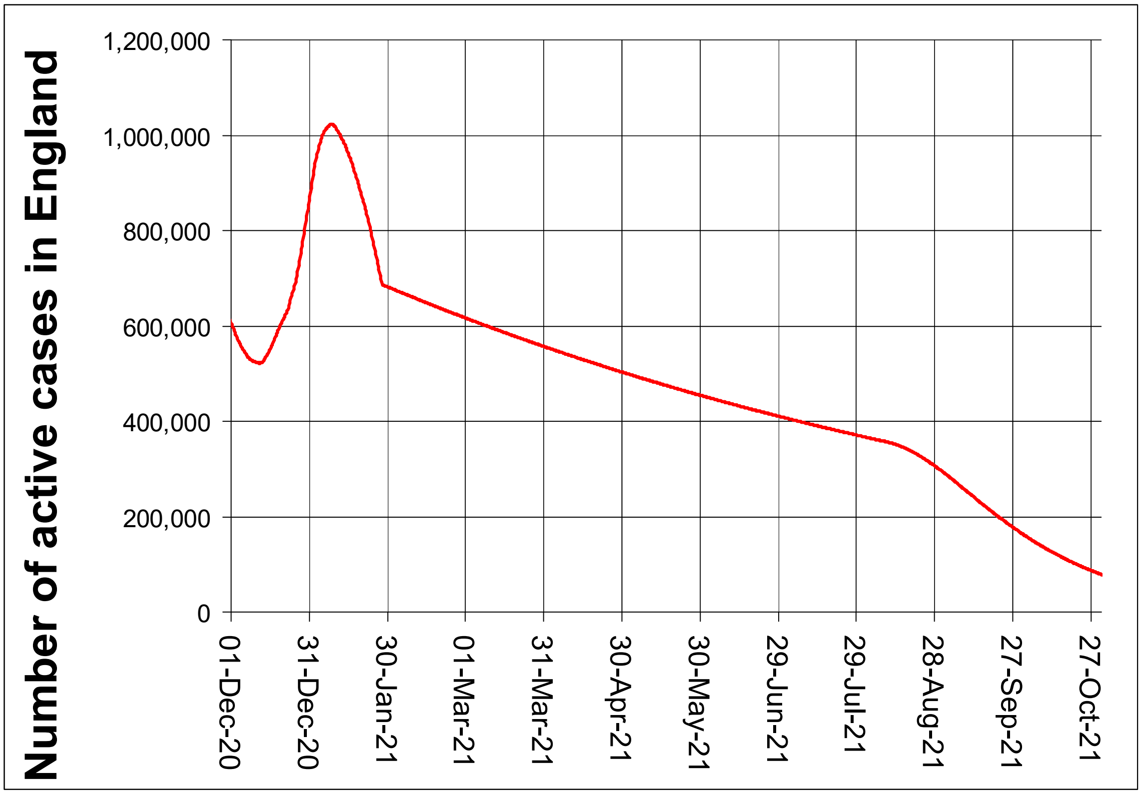

Figure 9. Active infections in England. *R* = 0.95

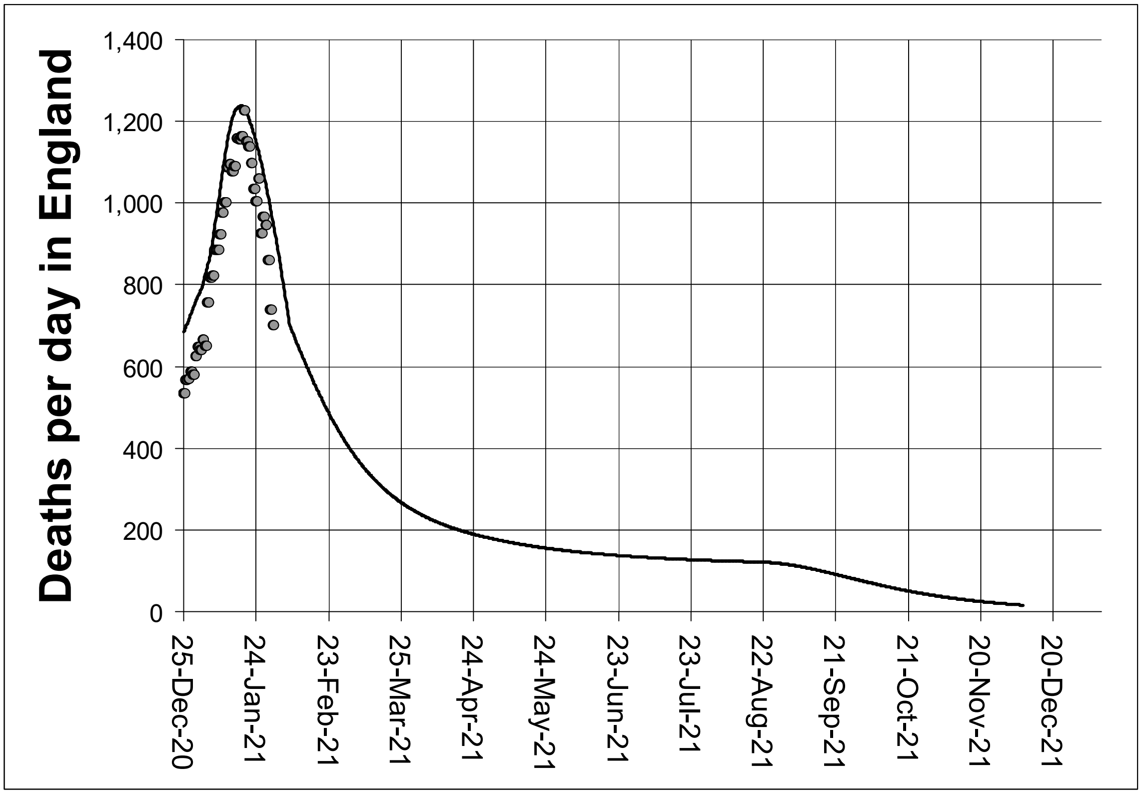

Figure 10. Deaths per day, *R* = 0.95

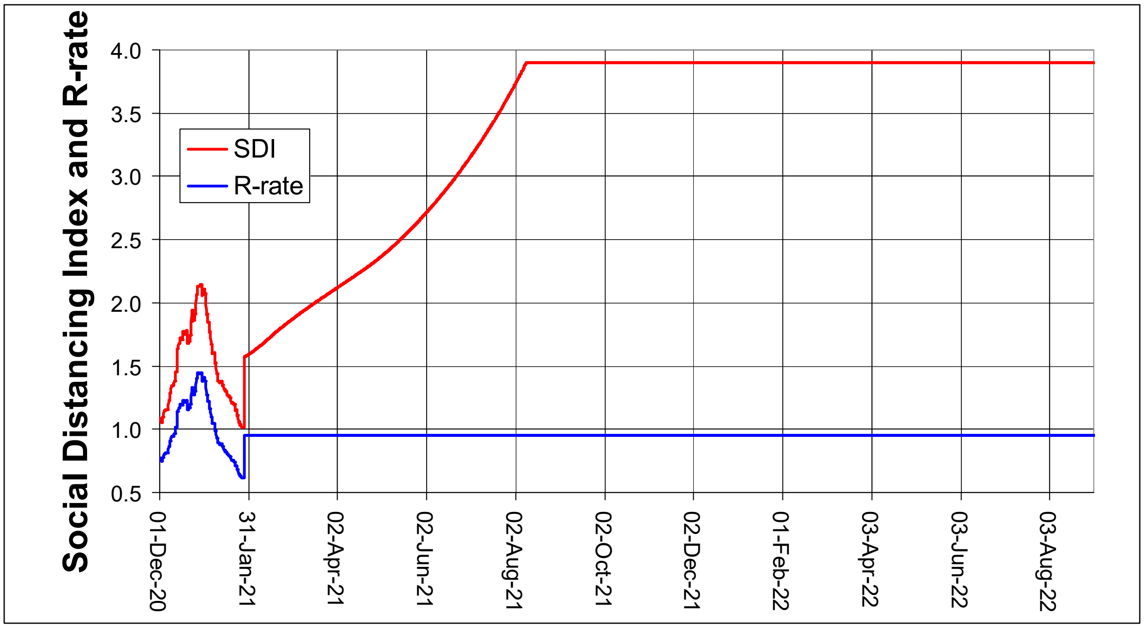

Figure 11. Social Distancing Index (SDI) and R-rate; *R* = 0.95

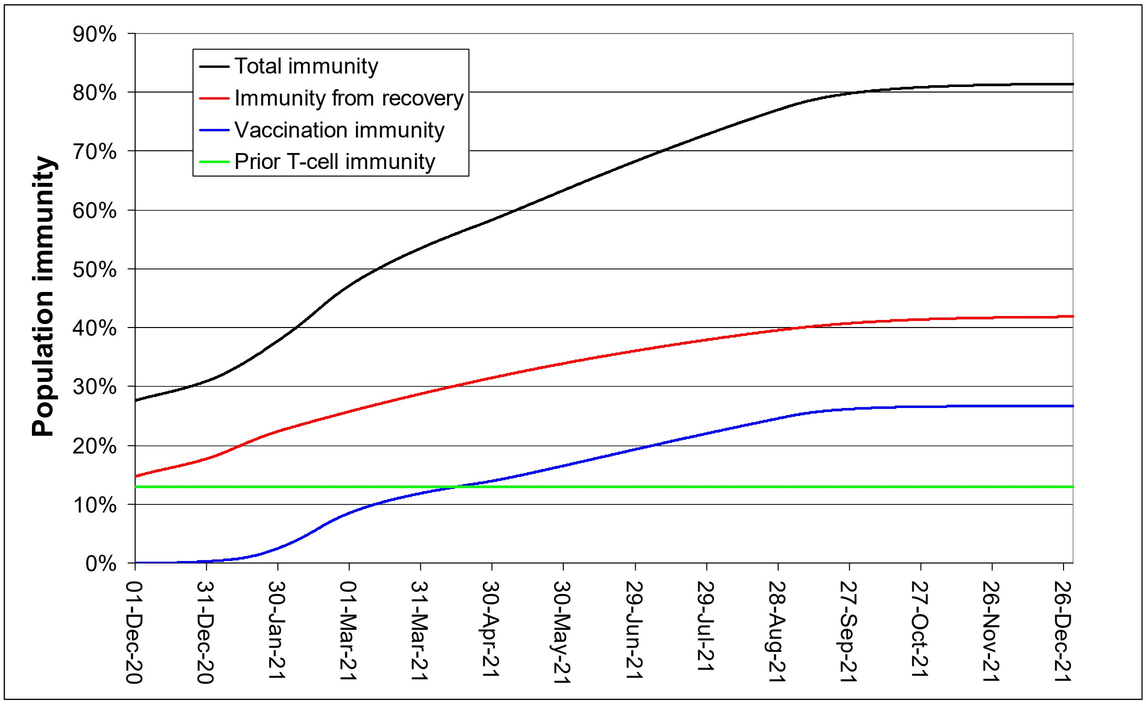

Figure 12. Components of population immunity, *R* = 0.95

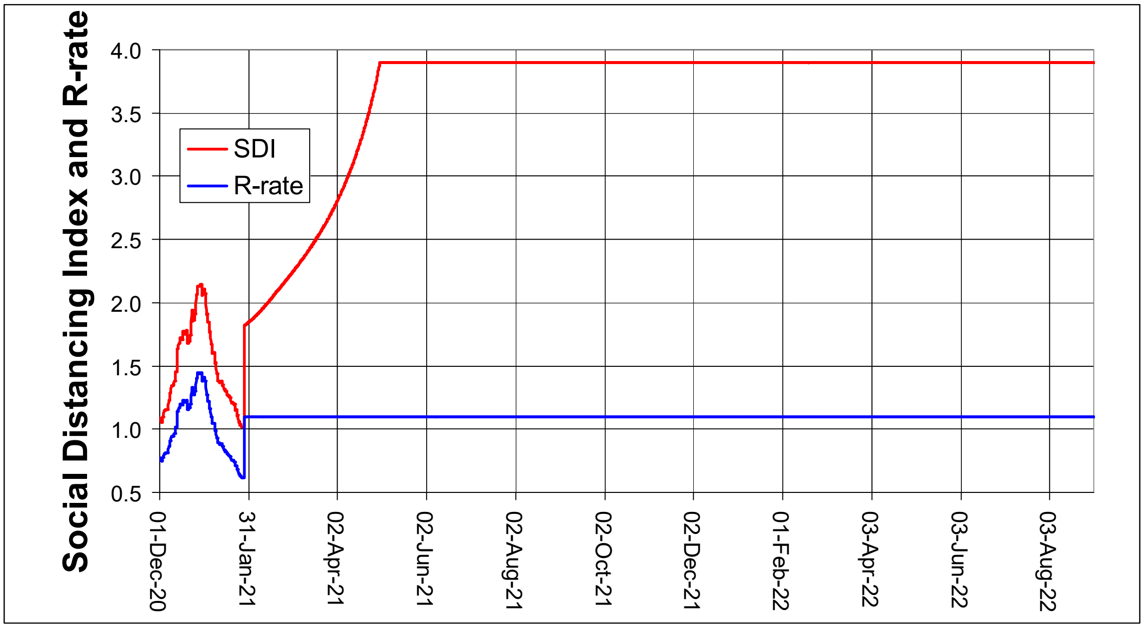

Figure 13. Social Distancing Index (SDI) and R-rate; *R* = 1.1

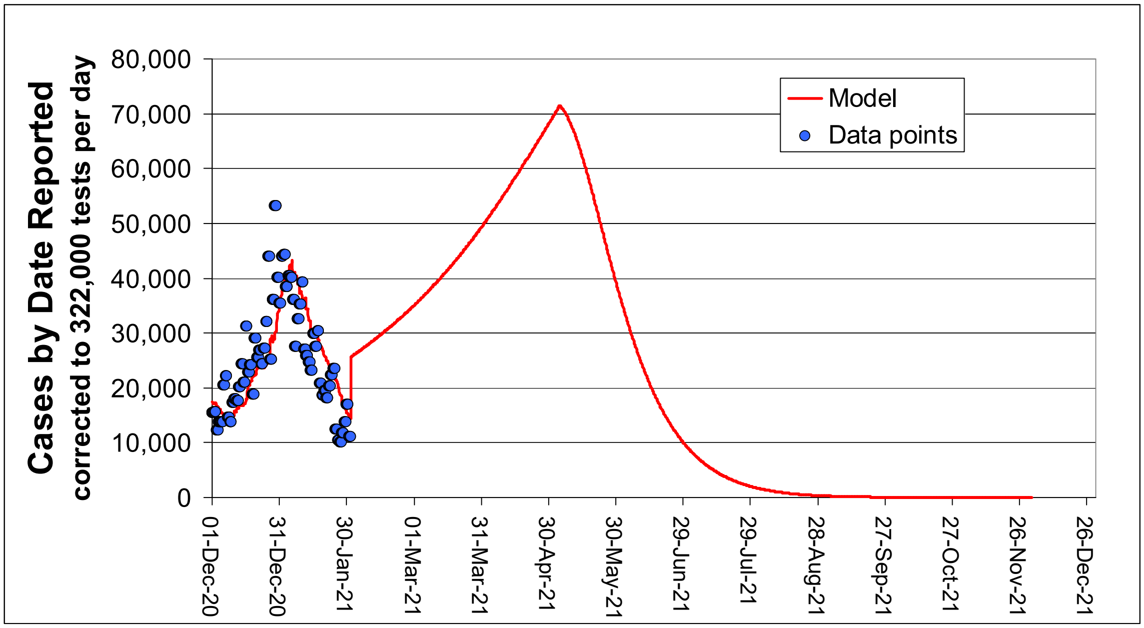

Figure 14. The effect on cases by date reported of keeping the R-rate at 1.1 from the end of January 2021 onwards

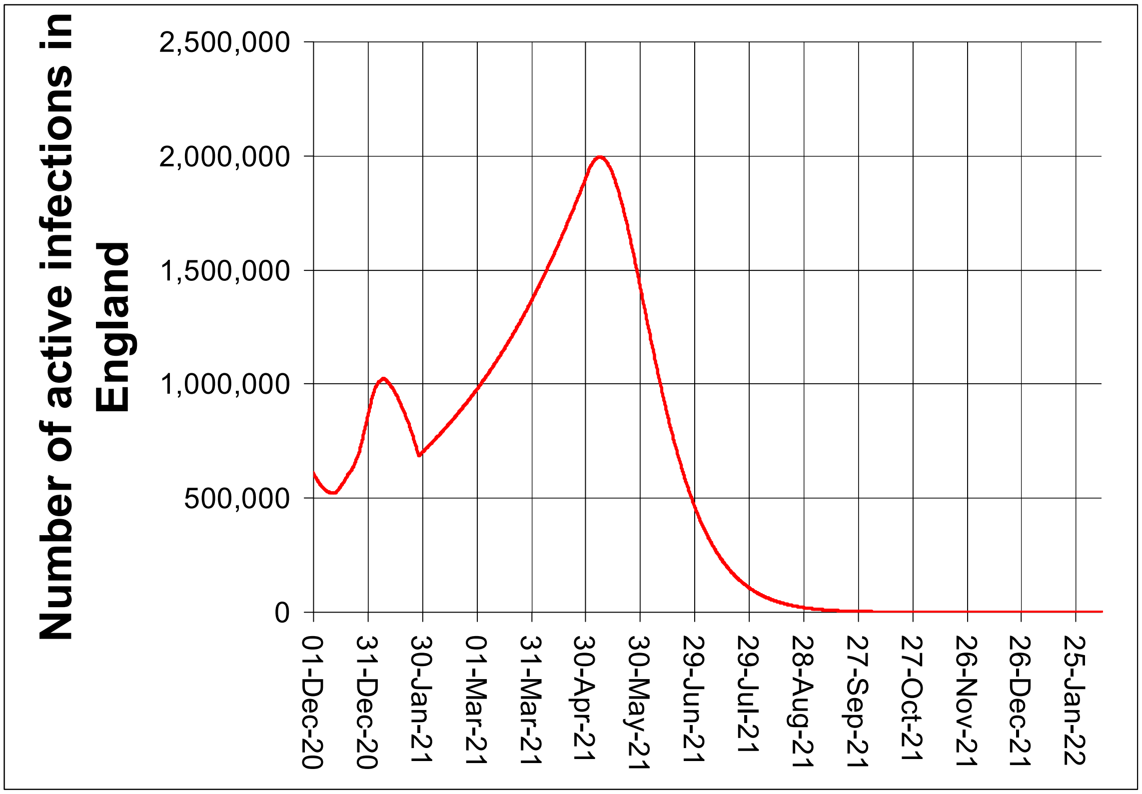

Figure 15. Active infections in England. *R* = 1.1

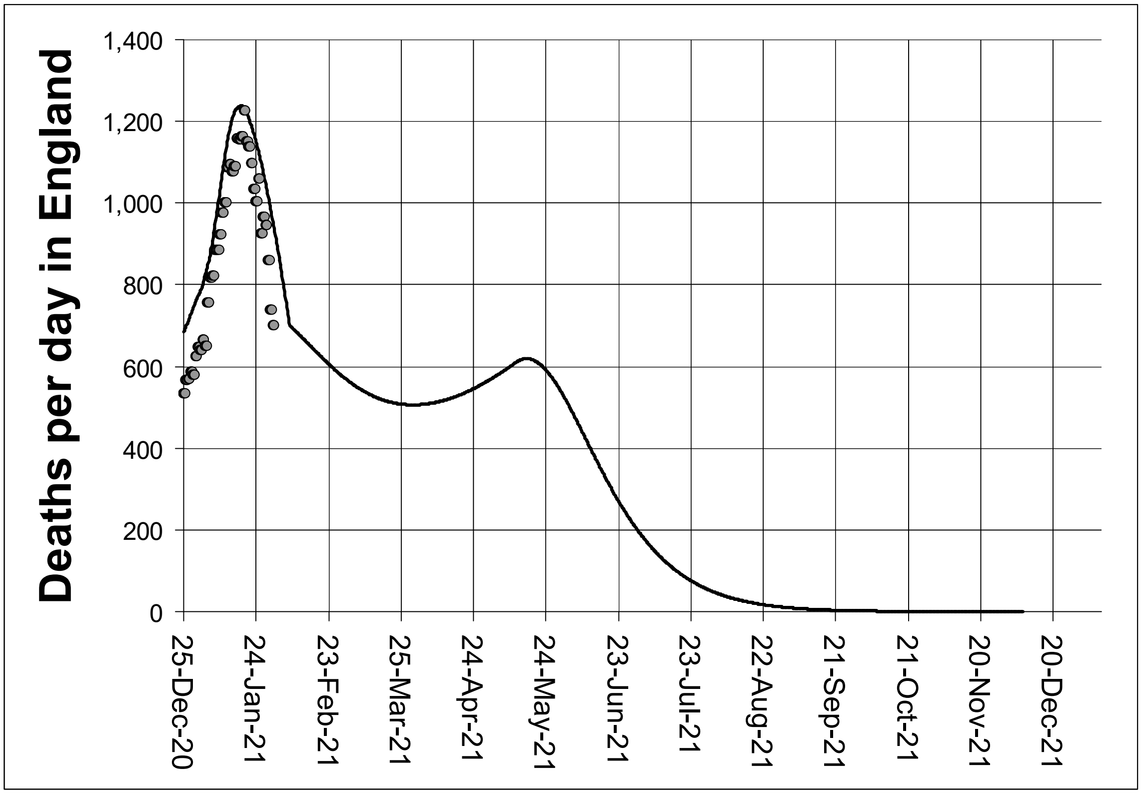

Figure 16. Daily deaths, *R* = 1.1

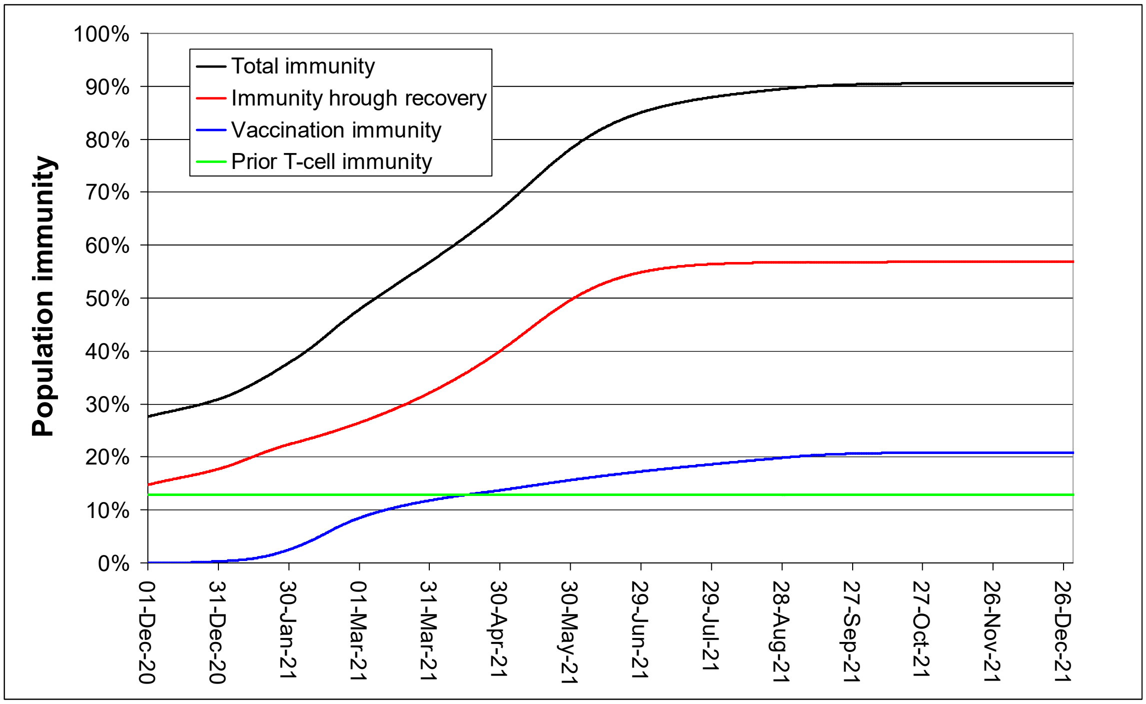

Figure 17. Components of population immunity, *R* = 1.1

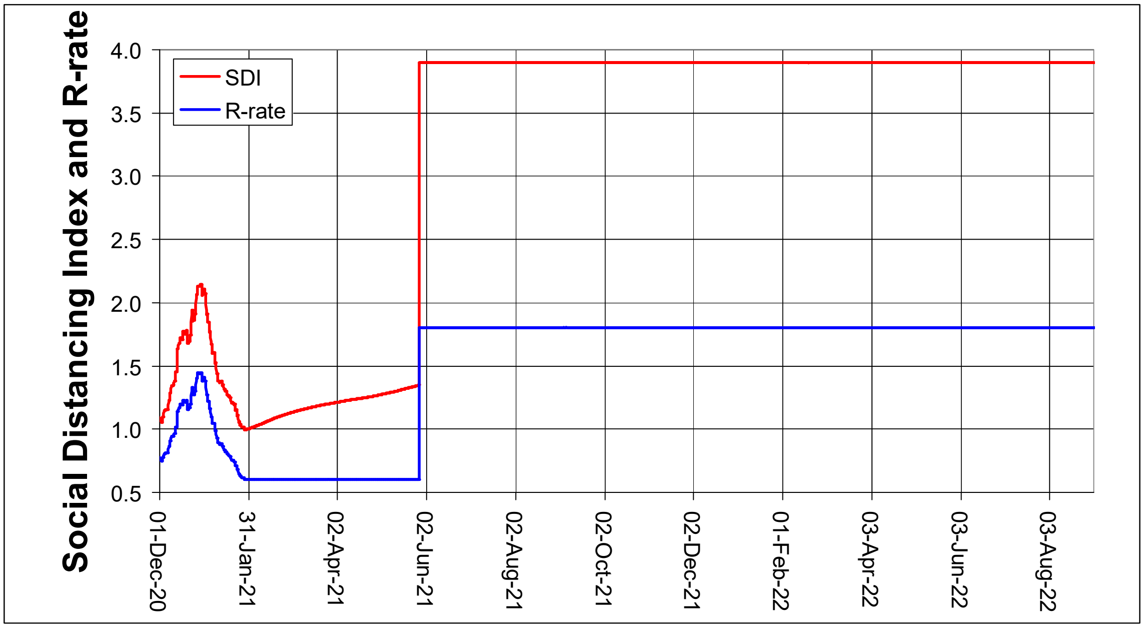

Figure 18. Social Distancing Index (SDI) and R-rate. Two-step strategy, R = 0.6 initially

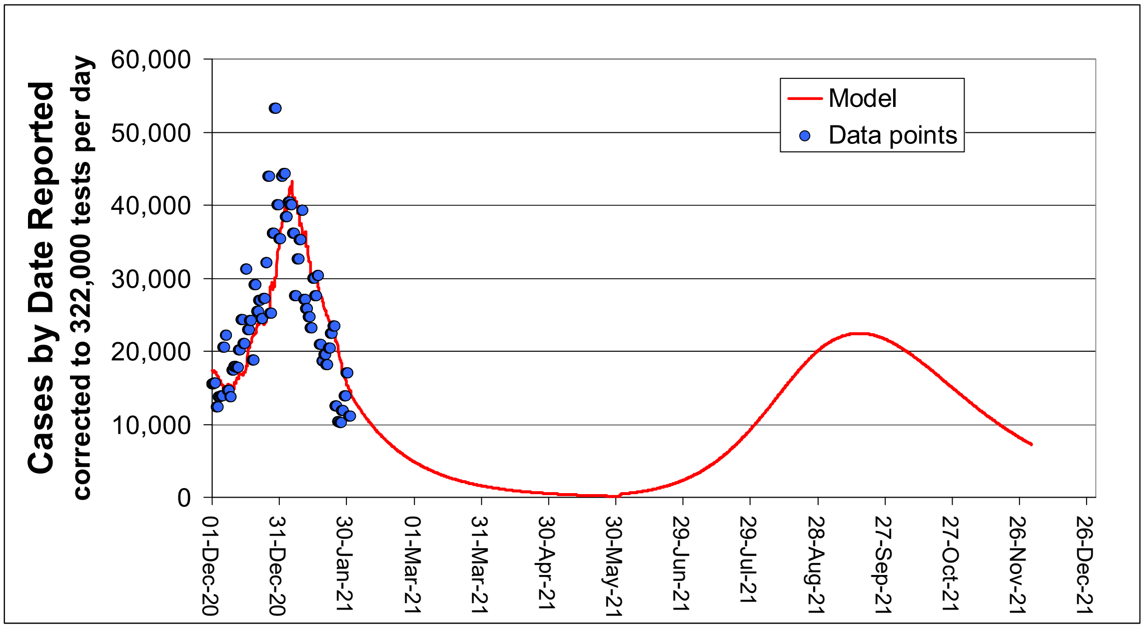

Figure 19. Cases by date reported. Two-step strategy, R = 0.6 initially

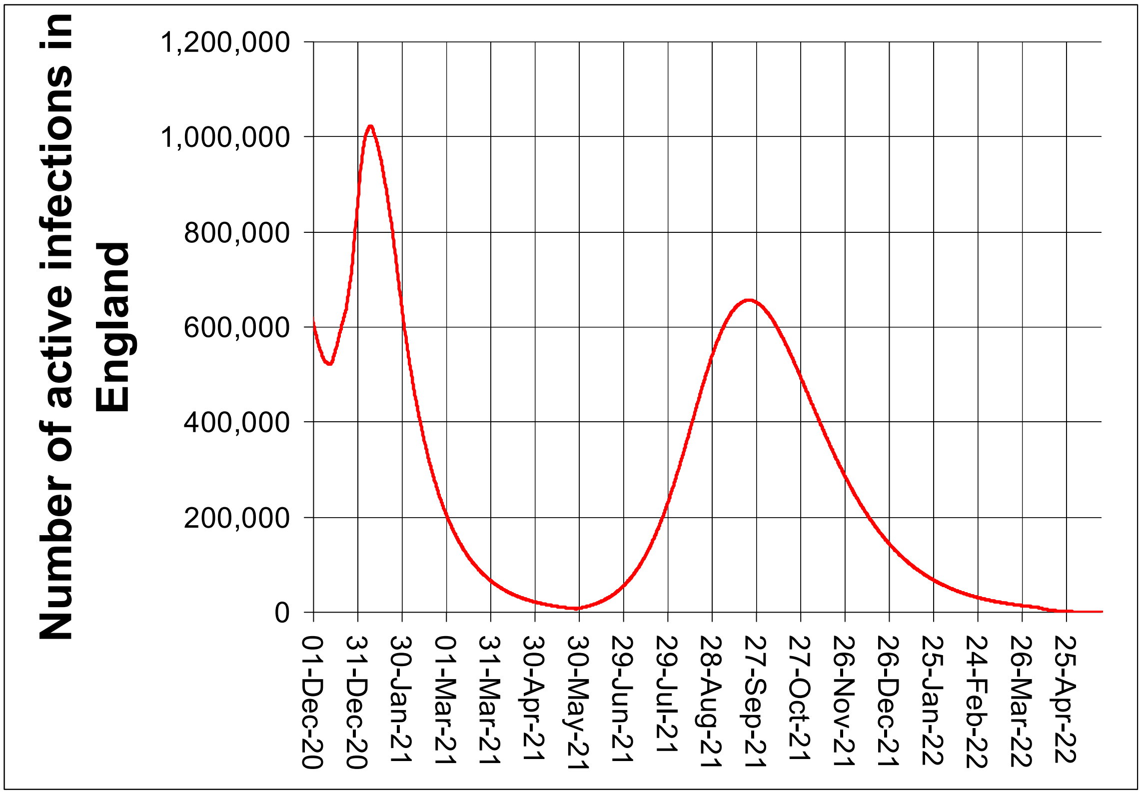

Figure 20. Active infections in England. Two-step strategy, R = 0.6 initially

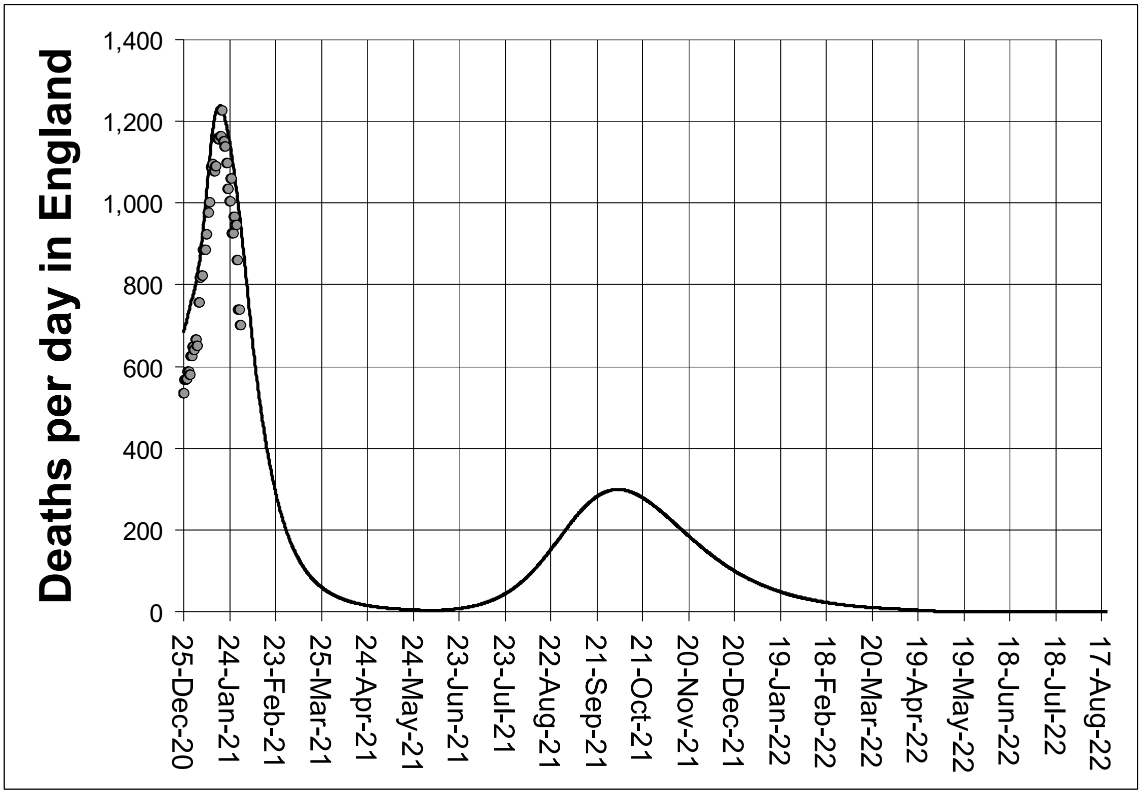

Figure 21. Daily deaths. Two-step strategy, R = 0.6 initially

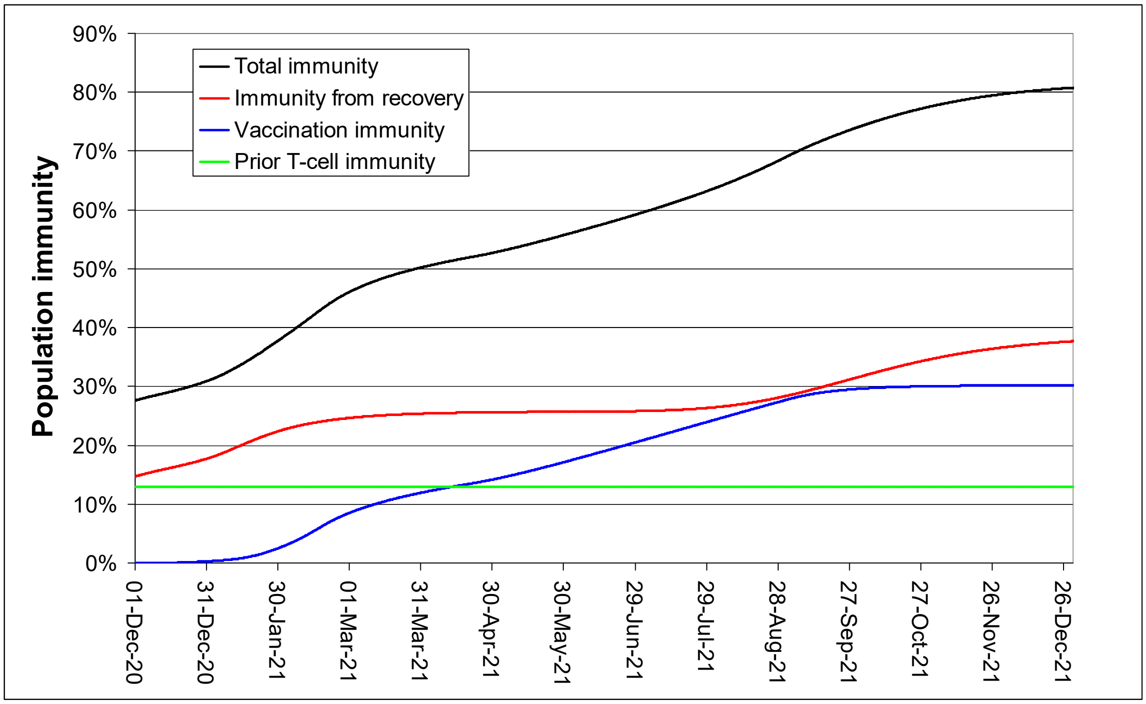

Figure 22. Components of population immunity. Two-step strategy, R = 0.6 initially

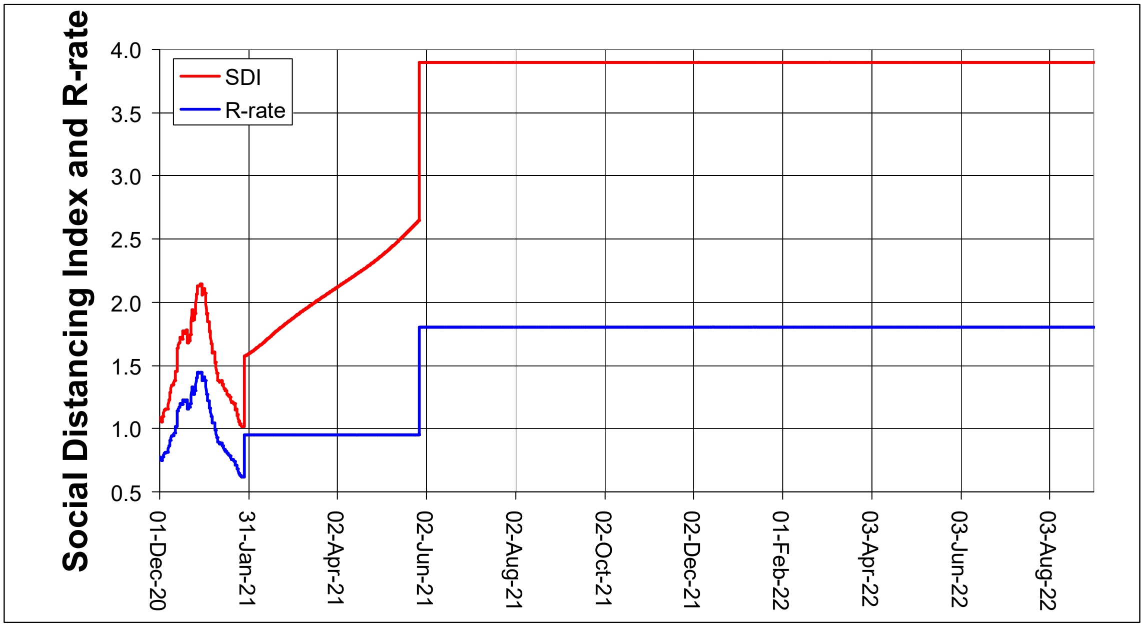

Figure 23. Social Distancing Index (SDI) and R-rate. Two-step strategy, R = 0.95 initially

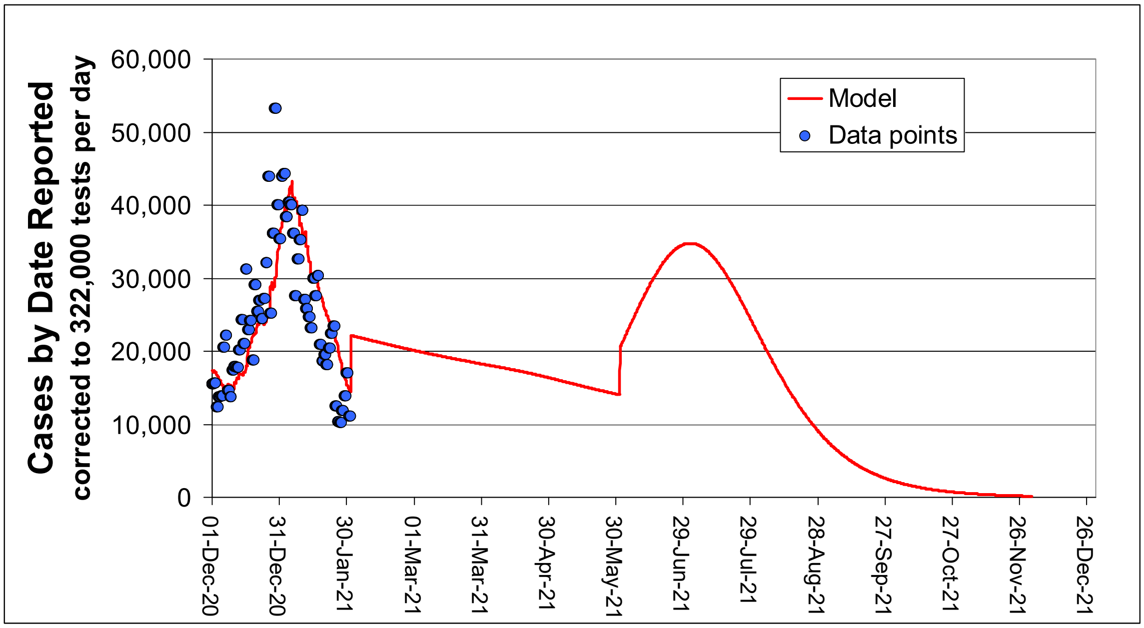

Figure 24. Cases by date reported. Two-step strategy, R = 0.95 initially

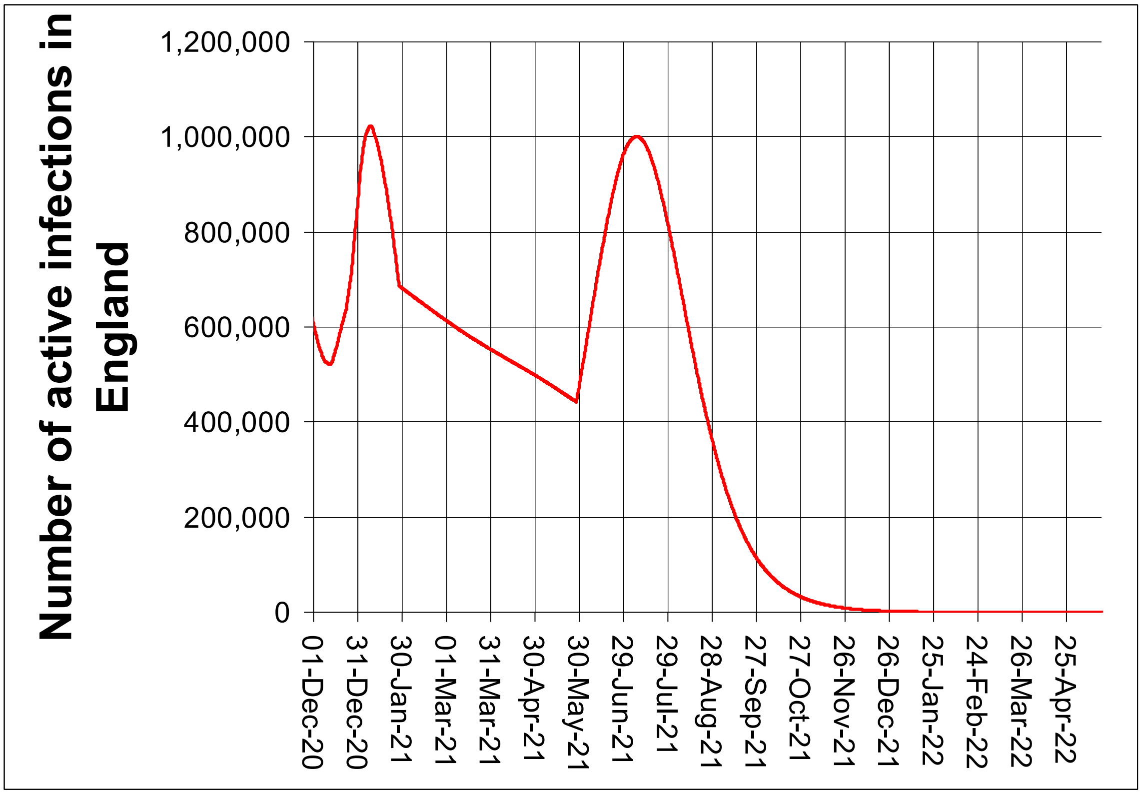

Figure 25. Active infections in England. Two-step strategy, R = 0.95 initially

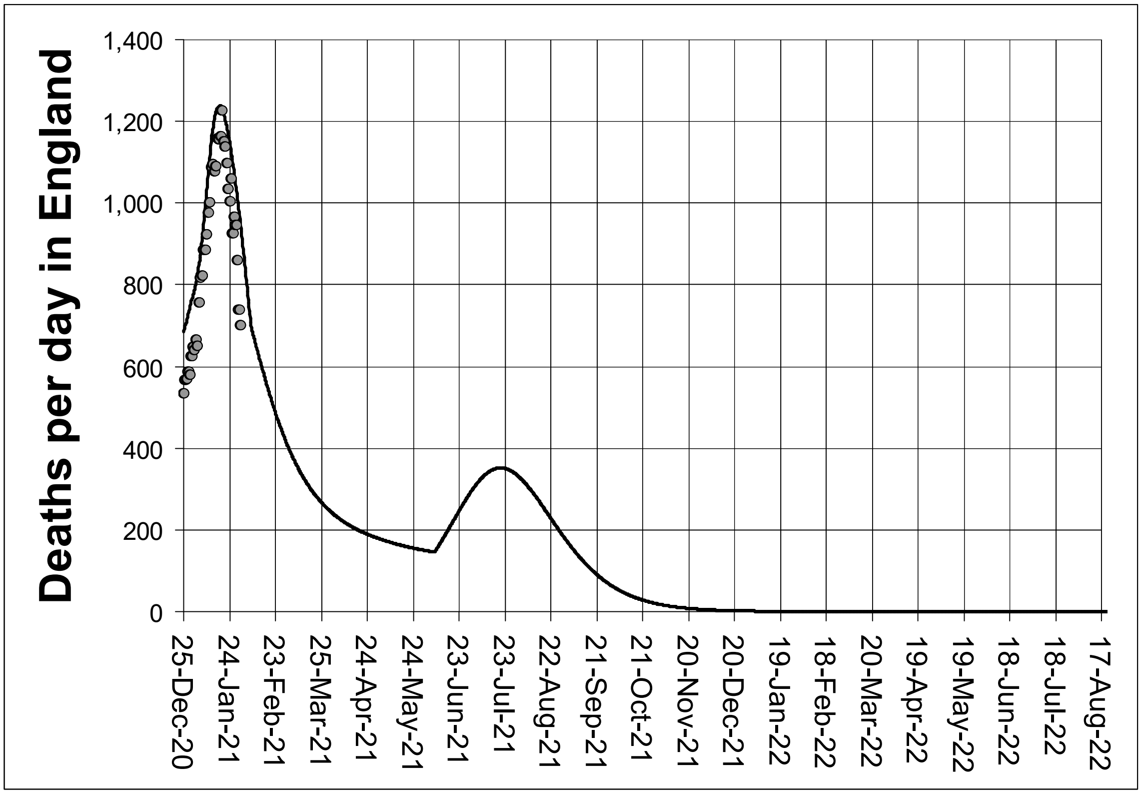

Figure 26. Daily deaths. Two-step strategy, R = 0.95 initially

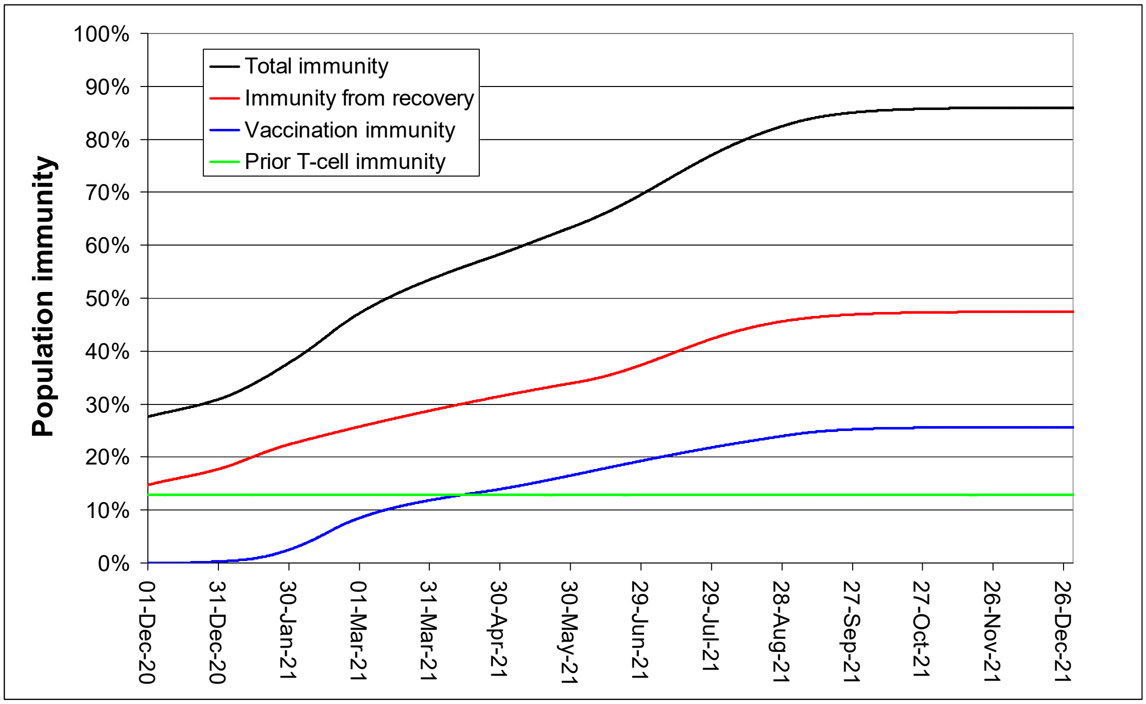

Figure 27. Components of population immunity. Two-step strategy, R = 0.95 initially

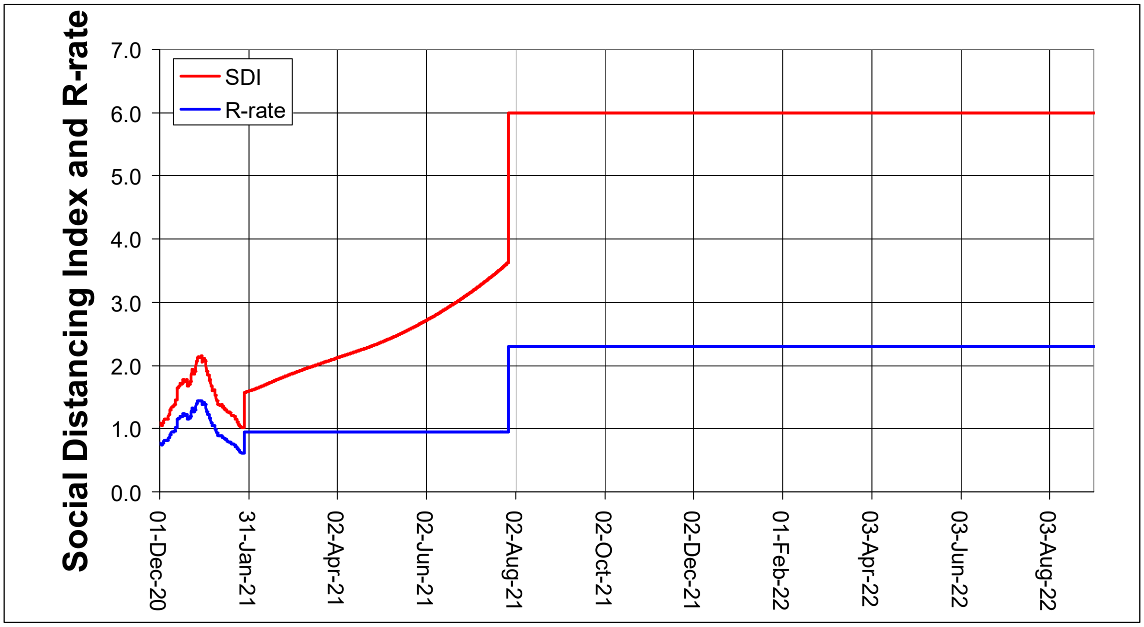

Figure 28. Social Distancing Index (SDI) and R-rate. Two-step strategy, R = 0.95 initially. Sensitivity study with $\Sigma_0 = 6$

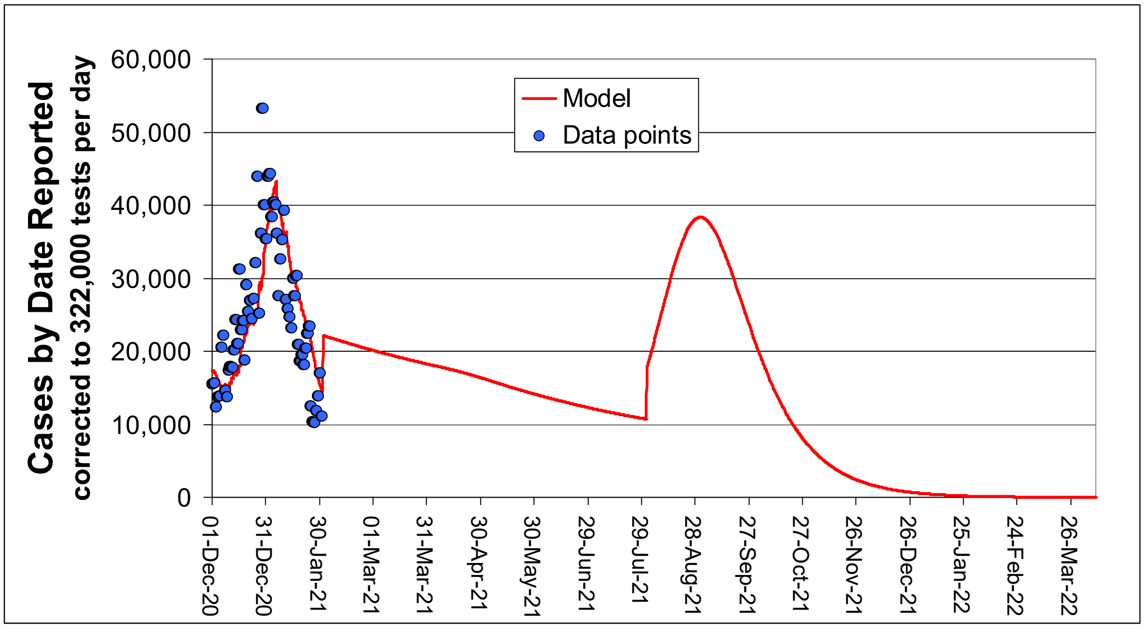

Figure 29. Cases by date reported. Two-step strategy, R = 0.95 initially. Sensitivity study with $\Sigma_0 = 6$

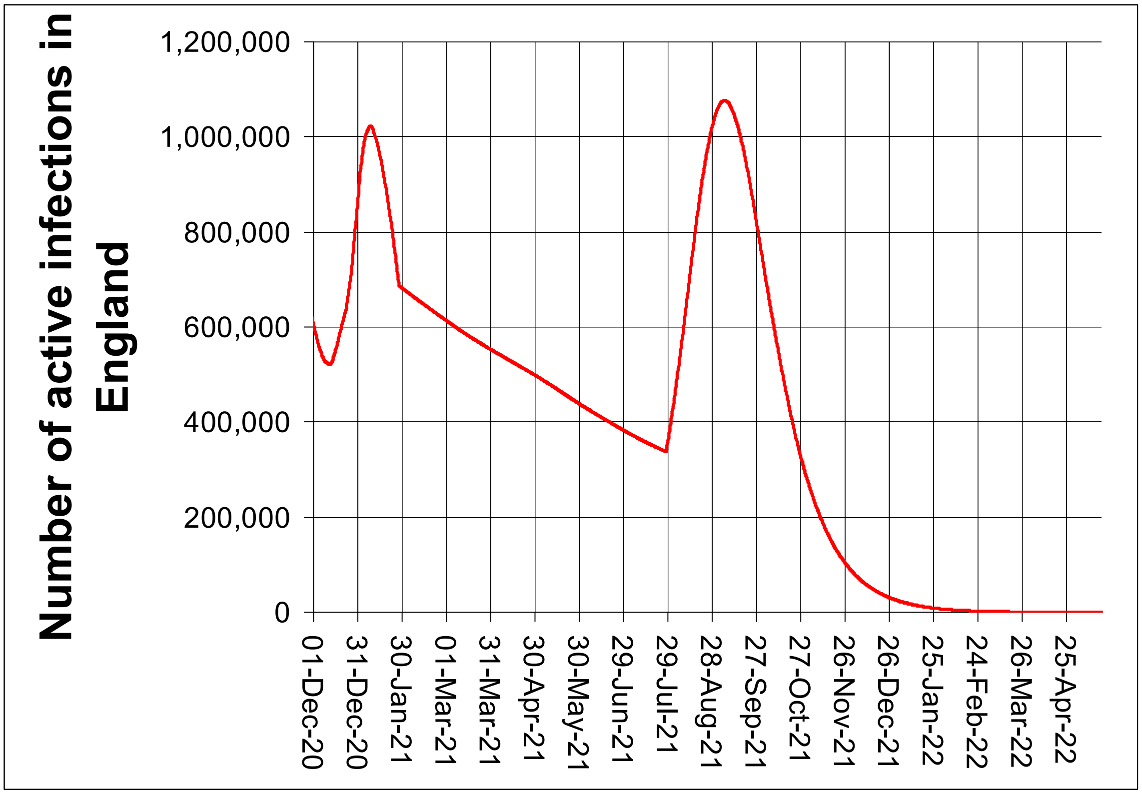

Figure 30. Active infections in England. Two-step strategy, R = 0.95 initially. Sensitivity study with $\Sigma_0 = 6$

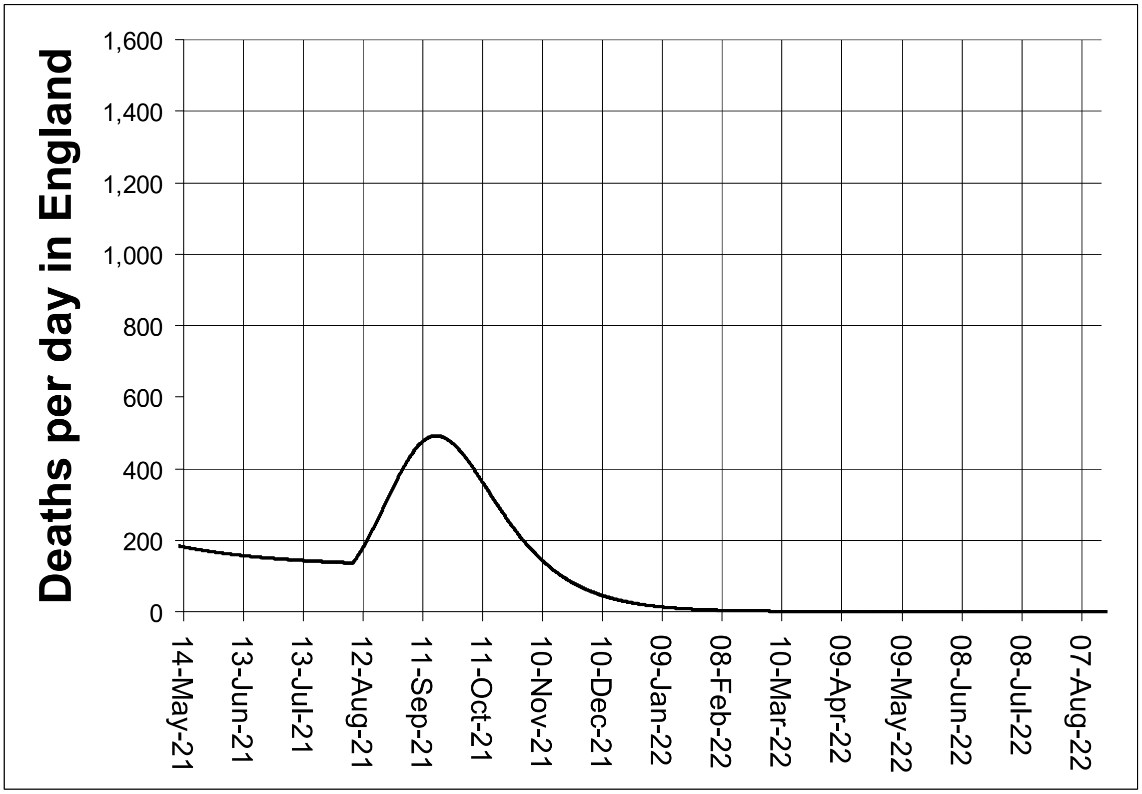

Figure 31. Daily deaths. Two-step strategy, R = 0.95 initially. Sensitivity study with $\Sigma_0 = 6$

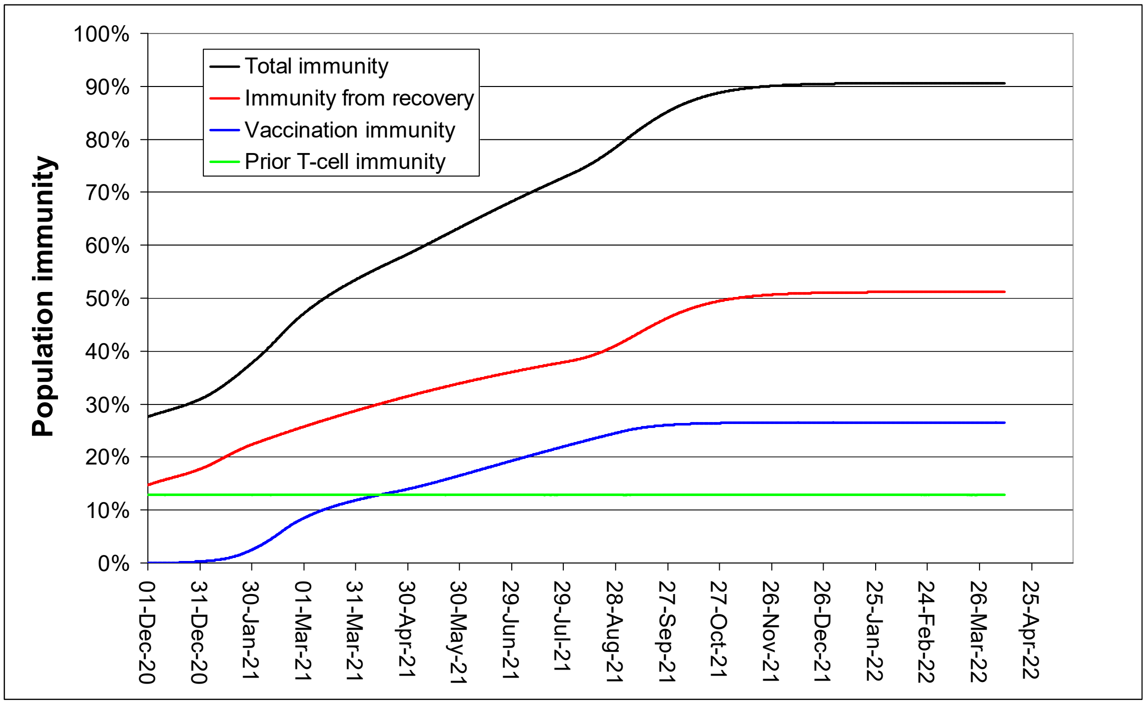

Figure 32. Components of population immunity. Two-step strategy, R = 0.95 initially. Sensitivity study with $\Sigma_0 = 6$

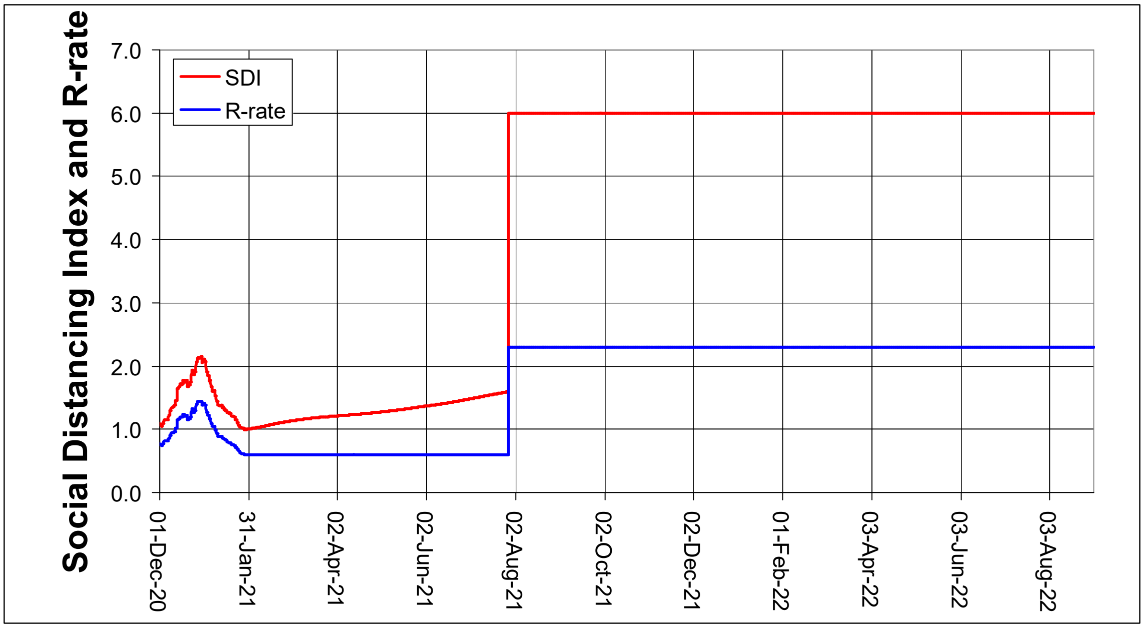

Figure 33. Social Distancing Index (SDI) and R-rate. Two-step strategy, R = 0.6 initially. Sensitivity study with $\Sigma_0 = 6$

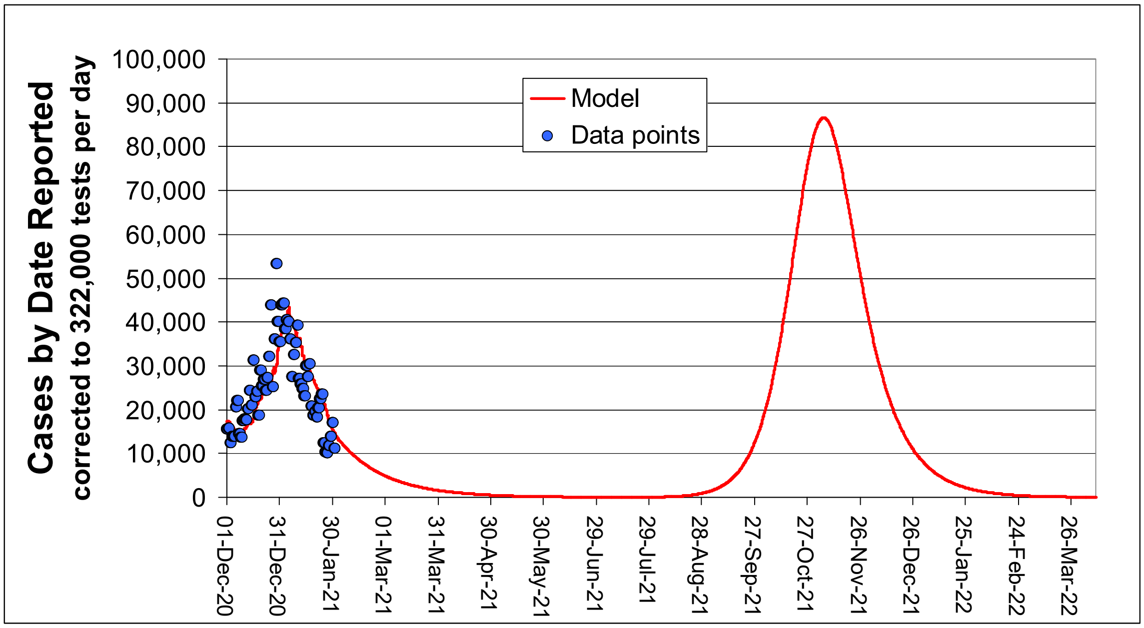

Figure 34. Cases by date reported. Two-step strategy, R = 0.6 initially. Sensitivity study with $\Sigma_0 = 6$

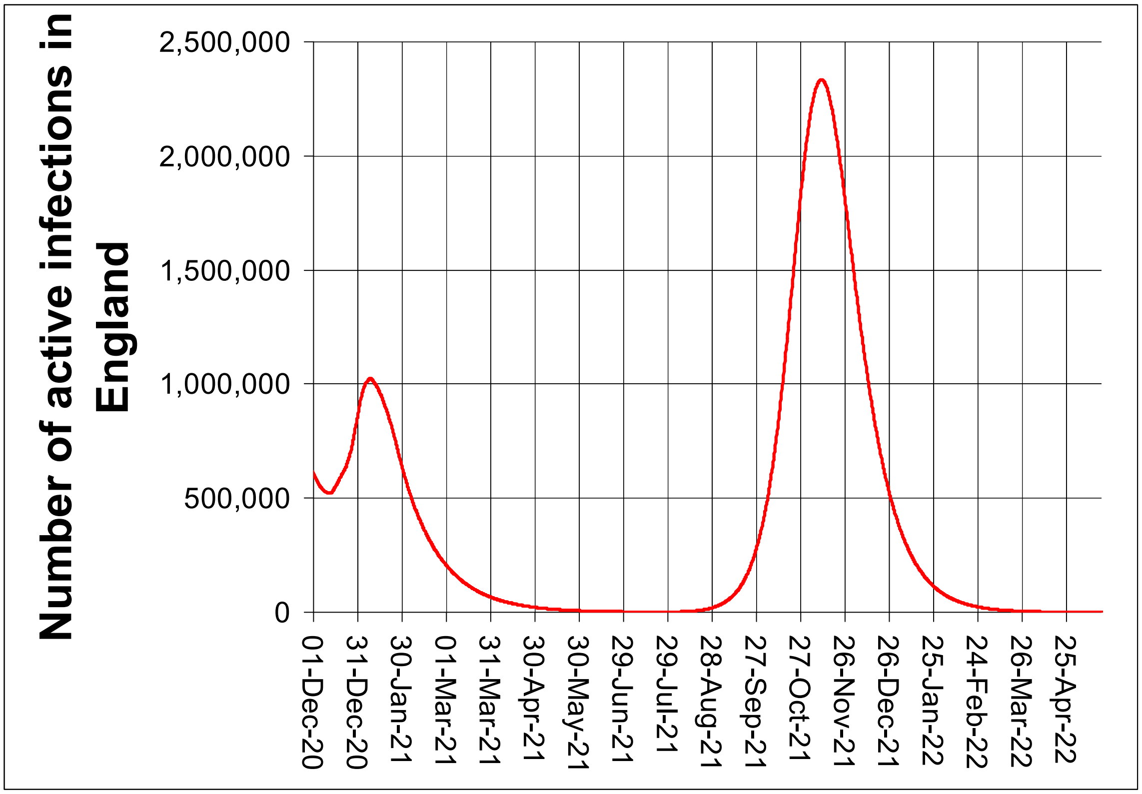

Figure 35. Active infections in England. Two-step strategy, R = 0.6 initially. Sensitivity study with $\Sigma_0 = 6$

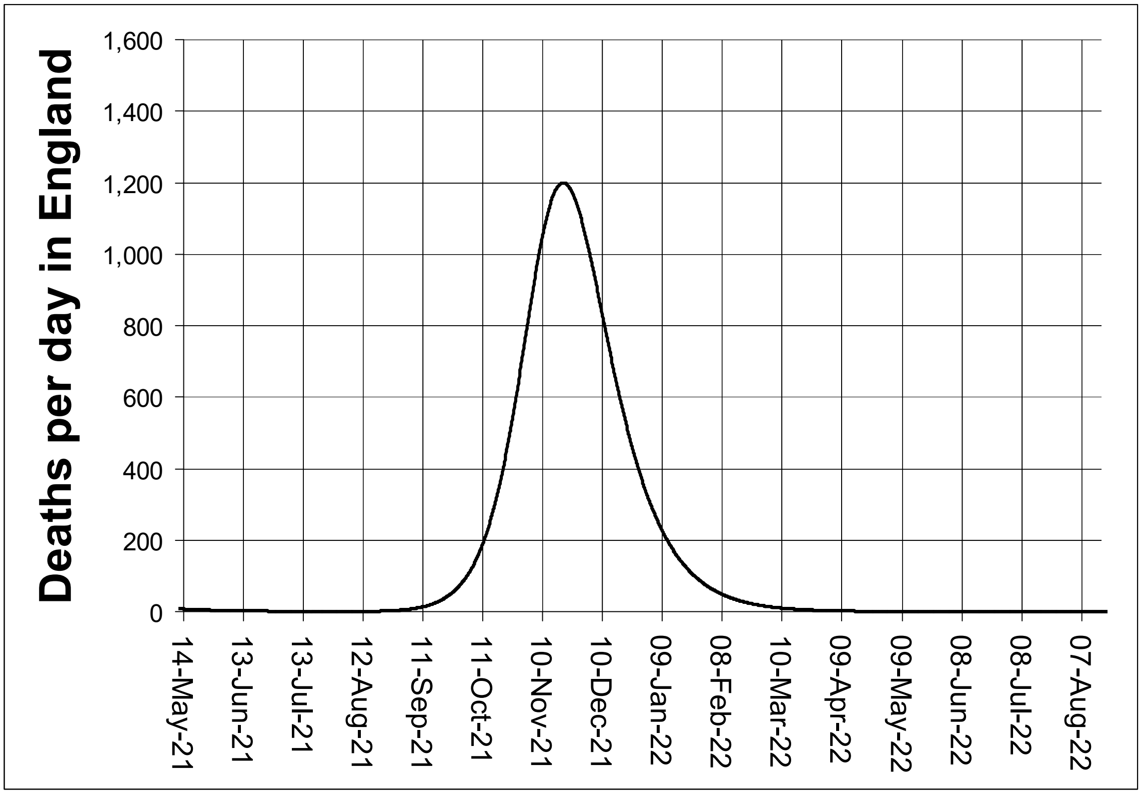

Figure 36. Daily deaths. Two-step strategy, R = 0.6 initially. Sensitivity study with $\Sigma_0 = 6$

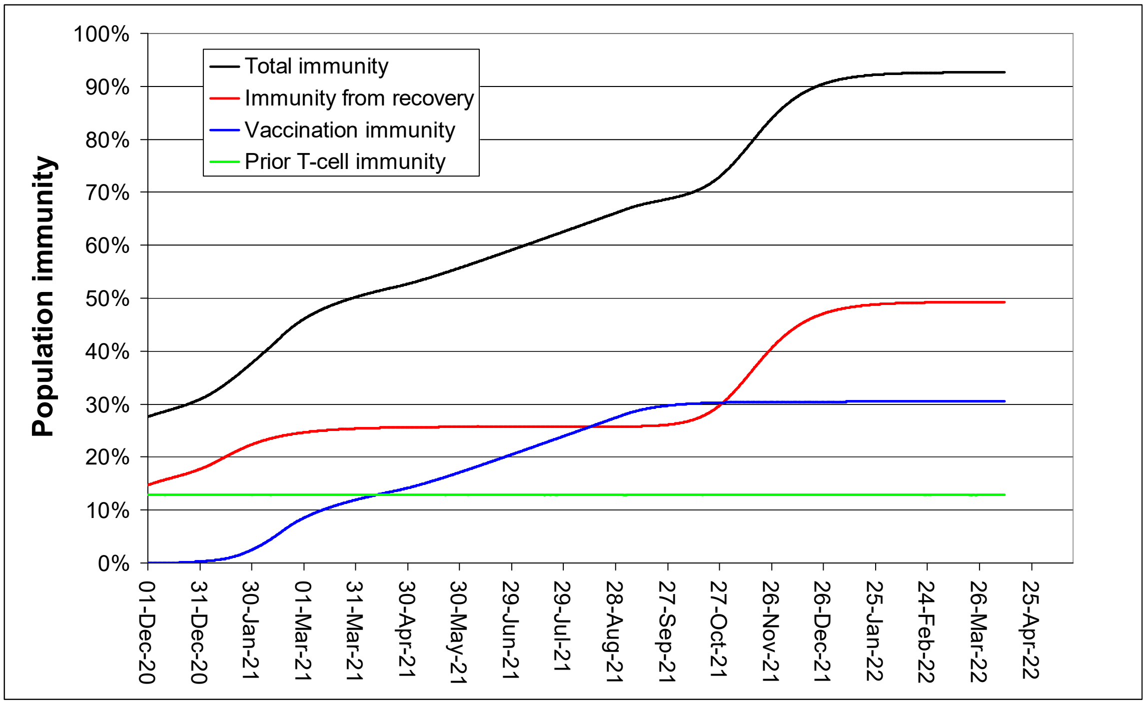

Figure 37. Components of population immunity. Two-step strategy, R = 0.6 initially. Sensitivity study with $\Sigma_0 = 6$